

The EKV MOSFET Transistor Small-signal Model

The Terminal Charges, Intrinsic Capacitances and Transcapacitances (Version 1)

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This notebook presents the derivation of the equivalent quasi-static (QS) small-signal circuit. This circuit extends the DC small-signal circuit including the transconductances to high frequency by adding the intrinsic capacitances and transcapacitances.

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1 Introduction

This notebook presents the derivation of the equivalent quasi-static (QS) small-signal circuit. This circuit extends the DC small-signal circuit including the transconductances to high frequency by adding the intrinsic capacitances and transcapacitances. In [1], we have derived this circuit from the non-quasi-static (NQS) model. This approach is rather complex, reason why in this notebook we will derive the QS model in a more standard way using the partitioning of the inversion channel charge between source and drain using the approach proposed by Ward and Dutton [2].

We start expressing the AC charging currents in terms of the intrinsic capacitances modeling the change of charges in the channel, on the gate terminal and depletion capacitance. The Ward and Dutton approach associates a charge to each terminal. The charge on the gate is easy to identify since it is equal to the total inversion charge in the channel and the fixed charges in the depletion capacitance. The charges associated to the source and drain terminals is a bit more tricky to define, but can be defined by partitioning the inversion charge in the channel between the source and the drain following the approach proposed by Ward and Dutton [2].

We then derive the terminal charges and express them in terms of the slope factor n , the normalized charges q_s and q_d at the source and drain. We then compute the derivatives with respect to the terminal voltages. We then derive the expressions of all the intrinsic capacitances in terms of n , q_s and q_d . Finally, we will derive an equivalent small-signal circuit that requires 5 capacitances and 3 transcapacitances.

2 Small-signal AC terminal charging currents

In this section we follow the small-signal model derivation done in [3] [4].

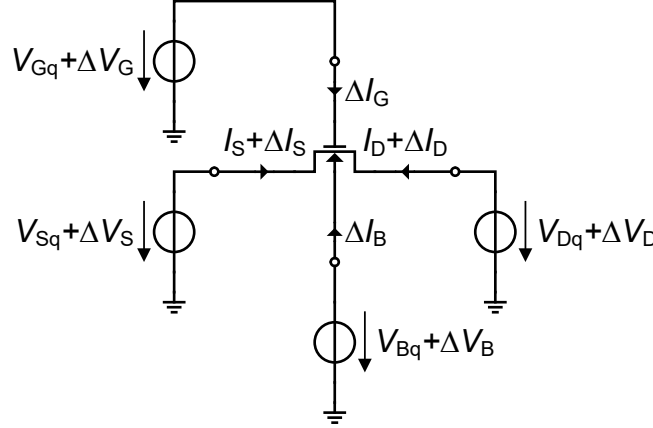


Figure 2.1: Definition of the voltages and currents [3] [4].

The MOSFET being a four terminal device, we can define the terminal voltages and currents as shown in Figure 2.1, where V_{Dq} , V_{Gq} , V_{Sq} and V_{Bq} are the DC operating point voltages and ΔV_D , ΔV_G , ΔV_S and ΔV_B are the small-signal voltage variations. $I_D = -I_S$ are the drain and source quiescent currents and ΔI_D , ΔI_G , ΔI_S and ΔI_B are the small-signal variations. Of course the currents need to satisfy the KCL. Neglecting the DC parts and considering only the AC components we can then write

$$\Delta I_D + \Delta I_G + \Delta I_S + \Delta I_B = 0. \quad (2.1)$$

The small-signal source and drain current variations include the change of DC current through the transconductances and the AC components due to the charging currents

$$\Delta I_D = \Delta I_{D,dc} + \Delta I_{D,ac}, \quad (2.2)$$

$$\Delta I_S = -\Delta I_{D,dc} + \Delta I_{S,ac}, \quad (2.3)$$

where $\Delta I_{D,dc}$ is the current change due to the transconductances and $\Delta I_{D,ac}$ and $\Delta I_{S,ac}$ are the AC charging currents due to the changes of charges Q_D and Q_S associated to the drain and source terminals.

Knowing that the charge variations only depend on ΔV_{DB} , ΔV_{GB} and ΔV_{SB} , we can write the

$$\Delta V_{DB} = \Delta V_D - \Delta V_B, \quad (2.4)$$

$$\Delta V_{GB} = \Delta V_G - \Delta V_B, \quad (2.5)$$

$$\Delta V_{SB} = \Delta V_S - \Delta V_B. \quad (2.6)$$

The small-signal terminal charging currents are given by [3] [4]

$$\Delta I_{D,ac}(t) = +C_{DD} \frac{dV_D}{dt} - C_{DG} \frac{dV_G}{dt} - C_{DS} \frac{dV_S}{dt} - C_{DB} \frac{dV_B}{dt}, \quad (2.7)$$

$$\Delta I_G(t) = -C_{GD} \frac{dV_D}{dt} + C_{GG} \frac{dV_G}{dt} - C_{GS} \frac{dV_S}{dt} - C_{GB} \frac{dV_B}{dt}, \quad (2.8)$$

$$\Delta I_{S,ac}(t) = -C_{SD} \frac{dV_D}{dt} - C_{SG} \frac{dV_G}{dt} + C_{SS} \frac{dV_S}{dt} - C_{SB} \frac{dV_B}{dt}. \quad (2.9)$$

$$\Delta I_B(t) = -C_{BD} \frac{dV_D}{dt} - C_{BG} \frac{dV_G}{dt} - C_{BS} \frac{dV_S}{dt} + C_{BB} \frac{dV_B}{dt}, \quad (2.10)$$

where the 16 intrinsic capacitances are defined by

$$C_{KK} = + \left. \frac{\partial Q_K}{\partial V_K} \right|_{op}, \quad (2.11)$$

$$C_{KL} = - \left. \frac{\partial Q_K}{\partial V_L} \right|_{op}. \quad (2.12)$$

where K and L are the terminals identifiers. Now, the 16 intrinsic capacitances are not independent. Indeed, because of the KCL, it can be shown that [3] [4]

$$C_{DD} = C_{DG} + C_{DS} + C_{DB} = C_{GD} + C_{SD} + C_{BD}, \quad (2.13)$$

$$C_{GG} = C_{GD} + C_{GS} + C_{GB} = C_{DG} + C_{SG} + C_{BG}, \quad (2.14)$$

$$C_{SS} = C_{SD} + C_{SG} + C_{SB} = C_{DS} + C_{GS} + C_{BS}, \quad (2.15)$$

$$C_{BB} = C_{BD} + C_{BG} + C_{BS} = C_{DB} + C_{GB} + C_{SB}. \quad (2.16)$$

For a bulk-referenced model like EKV, we have

$$V_D = V_{DB} + V_B, \quad (2.17)$$

$$V_G = V_{GB} + V_B, \quad (2.18)$$

$$V_S = V_{SB} + V_B, \quad (2.19)$$

and the small-signal terminal charging currents can then be written as

$$\Delta I_{D,ac}(t) = +C_{DD} \frac{dV_{DB}}{dt} - C_{DG} \frac{dV_{GB}}{dt} - C_{DS} \frac{dV_{SB}}{dt} + (C_{DD} - C_{DG} - C_{DS} - C_{DB}) \frac{dV_B}{dt}, \quad (2.20)$$

$$\Delta I_G(t) = -C_{GD} \frac{dV_{DB}}{dt} + C_{GG} \frac{dV_{GB}}{dt} - C_{GS} \frac{dV_{SB}}{dt} + (-C_{GD} + C_{GG} - C_{GB} - C_{GS}) \frac{dV_B}{dt}, \quad (2.21)$$

$$\Delta I_{S,ac}(t) = -C_{SD} \frac{dV_{DB}}{dt} - C_{SG} \frac{dV_{GB}}{dt} + C_{SS} \frac{dV_{SB}}{dt} + (-C_{SD} - C_{SG} - C_{SB} + C_{SS}) \frac{dV_B}{dt}. \quad (2.22)$$

$$(2.23)$$

Now, the last term of the above equations is actually equal to zero, resulting in

$$\Delta I_{D,ac}(t) = +C_{DD} \frac{dV_{DB}}{dt} - C_{DG} \frac{dV_{GB}}{dt} - C_{DS} \frac{dV_{SB}}{dt}, \quad (2.24)$$

$$\Delta I_G(t) = -C_{GD} \frac{dV_{DB}}{dt} + C_{GG} \frac{dV_{GB}}{dt} - C_{GS} \frac{dV_{SB}}{dt}, \quad (2.25)$$

$$\Delta I_{S,ac}(t) = -C_{SD} \frac{dV_{DB}}{dt} - C_{SG} \frac{dV_{GB}}{dt} + C_{SS} \frac{dV_{SB}}{dt}. \quad (2.26)$$

Replacing (2.13) in (2.24), (2.14) in (2.25) and (2.15) in (2.26) results in

$$\Delta I_{D,ac}(t) = (C_{BD} + C_{GD} + C_{SD}) \frac{dV_{DB}}{dt} - C_{DG} \frac{dV_{GB}}{dt} - C_{DS} \frac{dV_{SB}}{dt}, \quad (2.27)$$

$$\Delta I_G(t) = -C_{GD} \frac{dV_{DB}}{dt} + (C_{BG} + C_{DG} + C_{SG}) \frac{dV_{GB}}{dt} - C_{GS} \frac{dV_{SB}}{dt}, \quad (2.28)$$

$$\Delta I_{S,ac}(t) = -C_{SB} \frac{dV_{DB}}{dt} - C_{SG} \frac{dV_{GB}}{dt} + (C_{BS} + C_{DS} + C_{GS}) \frac{dV_{SB}}{dt}. \quad (2.29)$$

Note that the definition of the intrinsic capacitances given by (2.11) use the derivatives of the terminal charges with respect to an external reference. However, the terminal charges depend on V_{DB} , V_{GB} and V_{SB} . It is easy to show that the derivatives of the terminal charges with respect to the external reference are actually equal to the derivatives with respect to V_{DB} , V_{GB} and V_{SB} . For example, for $\partial Q_D / \partial V_G$, we have

$$C_{DG} = - \left. \frac{\partial Q_D}{\partial V_G} \right|_{V_D, V_S, V_B} = - \left. \frac{\partial Q_D}{\partial V_{GB}} \right|_{V_{DB}, V_{SB}} \cdot \left. \frac{\partial V_{GB}}{\partial V_G} \right|_{V_B} = - \left. \frac{\partial Q_D}{\partial V_{GB}} \right|_{V_{DB}, V_{SB}}. \quad (2.30)$$

We can therefore calculate the intrinsic capacitances by taking the derivatives wrt V_{DB} , V_{GB} and V_{SB} . The derivatives of the terminal charges are calculated in the next Section.

3 QS Terminal Charges

3.1 Total inversion charge

The total inversion charge in the channel is given by

$$Q_I = W \int_0^L Q_i(x) dx. \quad (3.1)$$

We can normalize Q_I as follows

$$q_I \triangleq \frac{Q_I}{W L Q_{spec}} = \int_0^1 q_i(\xi) d\xi \quad (3.2)$$

with $Q_{spec} \triangleq -2n C_{ox} U_T$ and

$$W L Q_{spec} \triangleq -2n W L C_{ox} U_T. \quad (3.3)$$

Similarly to what was done for calculating the drain current, we can perform the integration in the charge domain, remembering that the drain current is given by

$$i_d = -(2q_i + 1) \frac{dq_i}{d\xi}. \quad (3.4)$$

which gives

$$d\xi = -\frac{2q_i + 1}{i_d} dq_i. \quad (3.5)$$

Replacing $d\xi$ in (3.2) results in

$$q_I = \frac{1}{i_d} \int_{q_d}^{q_s} q_i (2q_i + 1) dq_i, \quad (3.6)$$

where $q_s \triangleq q_i(\xi = 0)$ and $q_d \triangleq q_i(\xi = 1)$.

Performing the integration leads to

$$q_I = \frac{4q_s^2 + 4q_d^2 + 4q_s q_d + 3q_s + 3q_d}{6i_d}. \quad (3.7)$$

Replacing the normalized drain current given by

$$i_d = (q_s^2 + q_s) - (q_d^2 + q_d) = q_s(q_s + 1) - q_s(q_d + 1) = (q_s - q_d)(q_s + q_s + 1), \quad (3.8)$$

results in

$$q_I = \frac{1}{6} \frac{4q_s^2 + 4q_d^2 + 4q_s q_d + 3q_s + 3q_d}{q_s + q_d + 1}. \quad (3.9)$$

In saturation (i.e. for $q_d = 0$) q_I reduces to

$$q_{I,sat} = \frac{q_s}{6} \frac{4q_s + 3}{(q_s + 1)} = \begin{cases} \frac{q_s}{2} & \text{in weak inversion } (q_s \ll 1), \\ \frac{2}{3} q_s & \text{in weak inversion } (q_s \gg 1). \end{cases} \quad (3.10)$$

3.2 Drain and source terminal charges

The total inversion charge needs to be partitioned between the drain and source terminals. Although this partitioning is a tricky question, under the assumption of constant mobility and uniform doping along the channel, the total inversion charge can be partitioned between the drain and source terminals according to the Ward and Dutton partitioning scheme given by [2] [3] [4]

$$Q_D = W \int_0^L \frac{x}{L} Q_i(x) dx, \quad (3.11)$$

$$Q_S = W \int_0^L \left(1 - \frac{x}{L}\right) Q_i(x) dx. \quad (3.12)$$

or in normalized form

$$q_D \triangleq \frac{Q_D}{W L Q_{spec}} = \int_0^1 \xi q_i(\xi) d\xi, \quad (3.13)$$

$$q_S \triangleq \frac{Q_S}{W L Q_{spec}} = \int_0^1 (1 - \xi) q_i(\xi) d\xi, \quad (3.14)$$

with $Q_{spec} \triangleq -2n C_{ox} U_T$ and

$$W L Q_{spec} \triangleq -2n W L C_{ox} U_T. \quad (3.15)$$

Caution

Carefull not to confuse q_S with q_s and q_D with q_d . q_s corresponds to the value of the normalized inversion charge at the source $q_s \triangleq q_i(\xi = 0)$, whereas q_S corresponds to the part of the total normalized inversion charge q_I that is associated to the source terminal. Similarly, q_d corresponds to the value of the normalized inversion charge at the drain $q_d \triangleq q_i(\xi = 1)$, while q_D is the part of the total normalized inversion charge that is associated to the drain terminal.

We can integrate (3.4) from to source to a position x (or ξ) along the channel

$$i_d \xi = \int_{q_i}^{q_s} (2q_i + 1) dq_i = (q_s^2 + q_s) - (q_i^2 + q_i) \quad (3.16)$$

resulting in

$$\xi = \frac{(q_s^2 + q_s) - (q_i^2 + q_i)}{i_d}. \quad (3.17)$$

Replacing the drain current by (3.4), results in

$$\xi = \frac{(q_s^2 + q_s) - (q_i^2 + q_i)}{(q_s^2 + q_s) - (q_d^2 + q_d)} = \frac{(q_s - q_i)(q_s + q_i + 1)}{(q_s - q_d)(q_s + q_d + 1)}. \quad (3.18)$$

The normalized drain charge can then be expressed as

$$q_D = \frac{1}{i_d^2} \int_{q_d}^{q_s} [(q_s^2 + q_s) - (q_i^2 + q_i)] q_i (2q_i + 1) dq_i. \quad (3.19)$$

Performing the integration results in

$$q_D = \frac{1}{60} \frac{16 q_s^3 + 24 q_d^3 + 32 q_s^2 q_d + 48 q_d^2 q_s + 25 q_s^2 + 45 q_d^2 + 50 q_s q_d + 10 q_s + 20 q_d}{q_s + q_d + 1} \quad (3.20)$$

In saturation (i.e. for $q_d = 0$) q_D reduces to

$$q_{D,sat} = \frac{q_s}{60} \frac{16 q_s^2 + 25 q_s + 10}{(q_s + 1)^2} = \begin{cases} \frac{q_s}{6} & \text{in weak inversion } (q_s \ll 1), \\ \frac{4}{15} q_s & \text{in weak inversion } (q_s \gg 1). \end{cases} \quad (3.21)$$

Proceeding in a similar way for q_S , we get

$$q_S = \frac{1}{60} \frac{16 q_d^3 + 24 q_s^3 + 32 q_d^2 q_s + 48 q_s^2 q_d + 25 q_d^2 + 45 q_s^2 + 50 q_s q_d + 10 q_d + 20 q_s}{q_s + q_d + 1} \quad (3.22)$$

The source terminal charge can of course be obtained from the drain terminal charge using the source-drain symmetry $q_S(q_s, q_d) = q_D(q_d, q_s)$.

In saturation (i.e. for $q_d = 0$) q_S reduces to

$$q_{S,sat} = \frac{q_s}{60} \frac{24 q_s^2 + 45 q_s + 20}{(q_s + 1)^2} = \begin{cases} \frac{q_s}{3} & \text{in weak inversion } (q_s \ll 1), \\ \frac{2}{5} q_s & \text{in strong inversion } (q_s \gg 1). \end{cases} \quad (3.23)$$

We can check that the sum of q_D and q_S is equal to the total charge in the channel q_I

$$q_I = q_D + q_S = \frac{1}{6} \frac{4 q_s^2 + 4 q_d^2 + 4 q_s q_d + 3 q_s + 3 q_d}{q_s + q_d + 1}, \quad (3.24)$$

which as expected corresponds to the results shown in (3.9).

3.3 Bulk terminal charge

The bulk charge is approximately given by [5] [1]

$$Q_B \cong -\Gamma_b C_{ox} W L \sqrt{\Psi_0 + V_P} - \frac{n-1}{n} Q_I \quad (3.25)$$

or in normalized form

$$q_B \triangleq \frac{Q_B}{-2n W L C_{ox} U_T} = \frac{\gamma_b}{2n} \sqrt{\psi_0 + v_p} - \frac{n-1}{n} q_I, \quad (3.26)$$

where

$$\gamma_b \triangleq \frac{\Gamma_b}{\sqrt{U_T}}, \quad (3.27)$$

$$\psi_0 \triangleq \frac{\Psi_0}{U_T}, \quad (3.28)$$

$$v_p \triangleq \frac{V_P}{U_T}. \quad (3.29)$$

3.4 Gate terminal charge

The gate terminal charge is simply obtained by charge conservation

$$q_G = -q_B - q_I = -q_B - q_S - q_D, \quad (3.30)$$

where we have ignored the fixed charges. We can write q_G replacing q_B by (3.26), resulting in

$$q_G = -\frac{1}{n} \left(\frac{\gamma_b}{2} \sqrt{\psi_0 + v_p} + q_I \right). \quad (3.31)$$

The terminal charges are plotted in saturation (i.e. for $q_d=0$) versus q_s in Figure 3.1 and versus IC in Figure 3.2. We see that the source, drain and total inversion charges are small in weak and moderate and that the bulk charge is dominating. This doesn't mean that the derivatives of the source, drain and gate terminal charges remain smaller than the derivatives of the bulk terminal charge.

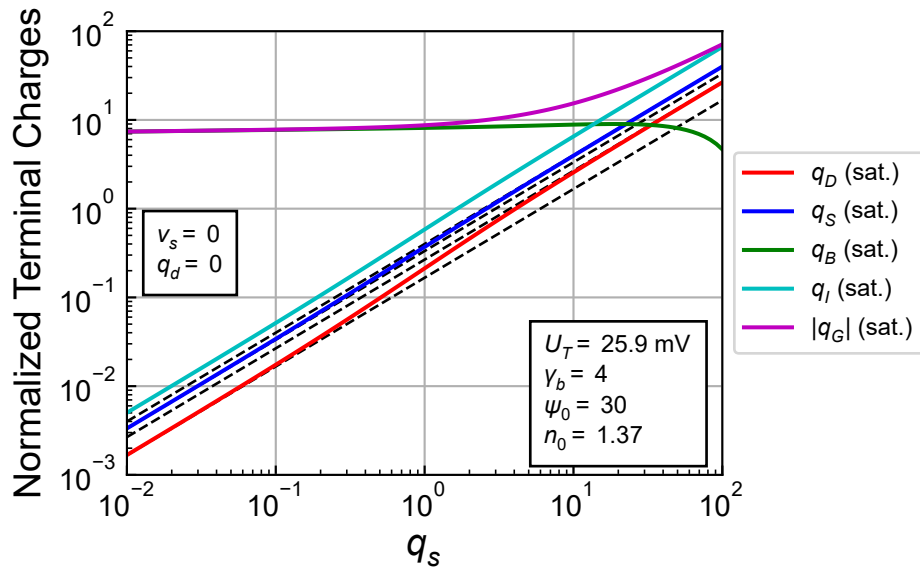


Figure 3.1: Terminal charges in saturation ($q_d = 0$) versus q_s in saturation ($q_d = 0$).

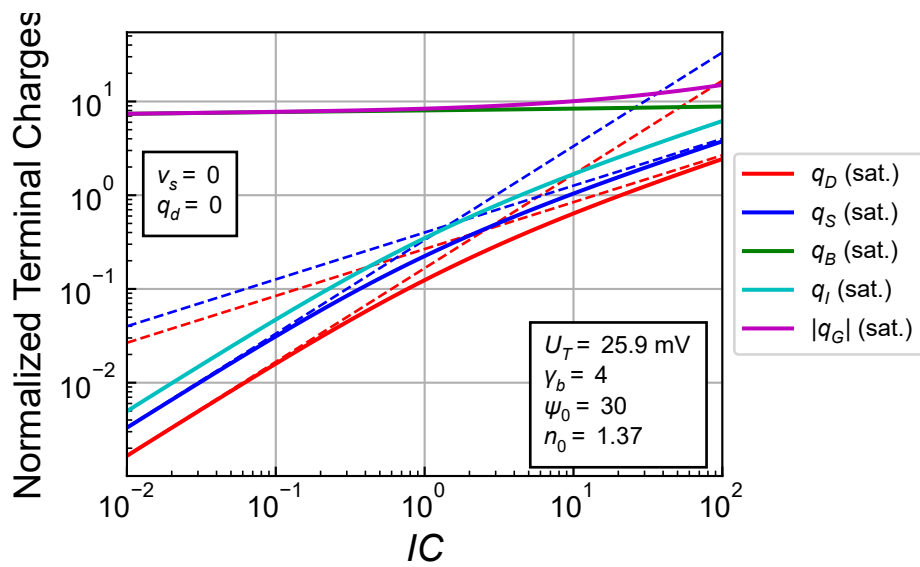


Figure 3.2: Terminal charges in saturation ($q_d = 0$) versus inversion factor IC .

4 Terminal charge derivatives

The purpose of this section is to write the derivatives of the total inversion charge q_I and the terminal charges q_S , q_D , q_B and q_G in terms of n , q_s and q_d only. We then need to compute only the derivatives of q_I , q_D and q_S wrt q_s and then get the derivatives of q_I , q_D and q_S wrt q_d by swapping q_s and q_d in the obtained derivatives.

i Note

To simplify the notation, in the following derivations, v_d , v_s and v_g stand actually for $v_{db} \triangleq V_{DB}/U_T$, $v_{gb} \triangleq V_{GB}/U_T$ and $v_{sb} \triangleq V_{SB}/U_T$.

! Important

The derivatives of the terminal charges with respect to the terminal voltages will be calculated assuming that the slope factor n is constant.

4.1 Derivatives of q_I

The derivatives of q_I wrt v_s and v_d are easy because q_I depends on q_s and q_d where q_s depends only on $v_p - v_s$ and not on v_d , whereas q_d depends only on $v_p - v_d$ and not on v_s . We can then use the chain rule and express the derivatives of q_I wrt v_d and v_s as

$$\frac{\partial q_I}{\partial v_d} = \frac{\partial q_I}{\partial q_d} \left(\frac{dv_d}{dq_d} \right)^{-1}, \quad (4.1)$$

$$\frac{\partial q_I}{\partial v_s} = \frac{\partial q_I}{\partial q_s} \left(\frac{dv_s}{dq_s} \right)^{-1}. \quad (4.2)$$

Recalling that $v_p - v_d$ and $v_p - v_s$ are related to q_d and q_s according to

$$v_p - v_d = 2q_d + \ln(q_d), \quad (4.3)$$

$$v_p - v_s = 2q_s + \ln(q_s), \quad (4.4)$$

we get

$$v_d = v_p - 2q_d - \ln(q_d), \quad (4.5)$$

$$v_s = v_p - 2q_s - \ln(q_s), \quad (4.6)$$

resulting in

$$\frac{dv_d}{dq_d} = -\frac{2q_d + 1}{q_d}, \quad (4.7)$$

$$\frac{dv_s}{dq_s} = -\frac{2q_s + 1}{q_s}. \quad (4.8)$$

The derivatives of q_I wrt v_d and v_s then write

$$\frac{\partial q_I}{\partial v_d} = -\frac{q_d}{2q_d + 1} \frac{\partial q_I}{\partial q_d} = -\frac{q_d}{6} \frac{2q_d + 4q_d + 3}{(q_s + q_d + 1)^2}, \quad (4.9)$$

$$\frac{\partial q_I}{\partial v_s} = -\frac{q_s}{2q_s + 1} \frac{\partial q_I}{\partial q_s} = -\frac{q_s}{6} \frac{2q_s + 4q_d + 3}{(q_s + q_d + 1)^2}. \quad (4.10)$$

When differentiating wrt v_g , we need to account for the fact that both q_s and q_d depend on v_p and therefore on v_g according to

$$v_p = \frac{v_g - v_{t0}}{n}. \quad (4.11)$$

The derivative with respect to v_g can then be written as

$$\frac{dq_I}{dv_g} = \frac{dq_I}{dv_p} \frac{dv_p}{dv_g} = \frac{1}{n} \frac{dq_I}{dv_p} \quad (4.12)$$

where

$$\frac{dq_I}{dv_p} = \frac{\partial q_I}{\partial q_d} \left(\frac{dv_p}{dq_d} \right)^{-1} + \frac{\partial q_I}{\partial q_s} \left(\frac{dv_p}{dq_s} \right)^{-1}, \quad (4.13)$$

and

$$\frac{dv_p}{dq_d} = \frac{2q_d + 1}{q_d}, \quad (4.14)$$

$$\frac{dv_p}{dq_s} = \frac{2q_s + 1}{q_s}. \quad (4.15)$$

The derivative of q_I wrt v_p is then given by

$$\frac{dq_I}{dv_p} = \frac{1}{6} \frac{2q_s^2 + 2q_d^2 + 8q_s q_d + 3q_s + 3q_d}{(q_s + q_d + 1)^2} \quad (4.16)$$

and finally the derivative of q_I wrt v_g by

$$\frac{dq_I}{dv_g} = \frac{1}{6n} \frac{2q_s^2 + 2q_d^2 + 8q_s q_d + 3q_s + 3q_d}{(q_s + q_d + 1)^2}. \quad (4.17)$$

Note that the derivative of q_I wrt v_p and v_g are symmetrical with respect to the source and drain. Indeed, we can swap q_s and q_d and the expressions (4.16) and (4.17) do not change.

4.2 Derivatives of q_D

Similarly to what was done above for differentiating q_I , the derivatives of q_D wrt v_d and v_s can be expressed as

$$\frac{\partial q_D}{\partial v_d} = \frac{\partial q_D}{\partial q_d} \left(\frac{dv_d}{dq_d} \right)^{-1} = -\frac{q_d}{2q_d + 1} \frac{\partial q_D}{\partial q_d}, \quad (4.18)$$

$$\frac{\partial q_D}{\partial v_s} = \frac{\partial q_D}{\partial q_s} \left(\frac{dv_s}{dq_s} \right)^{-1} = -\frac{q_s}{2q_s + 1} \frac{\partial q_D}{\partial q_s}. \quad (4.19)$$

Performing the differentiation results in

$$\frac{\partial q_D}{\partial v_d} = -\frac{q_d}{30} \frac{16q_s^2 + 6q_d^2 + 18q_s q_d + 25q_s + 15q_d + 10}{(q_s + q_d + 1)^3}, \quad (4.20)$$

$$\frac{\partial q_D}{\partial v_s} = -\frac{q_s}{30} \frac{4q_s^2 + 4q_d^2 + 12q_s q_d + 10q_s + 10q_d + 5}{(q_s + q_d + 1)^3}. \quad (4.21)$$

i Note

Notice that the expressions of $\partial q_D/\partial v_d$ and $\partial q_D/\partial v_s$ are not symmetrical wrt source and drain.

The derivative of q_D with respect to v_g is given by

$$\frac{dq_D}{dv_g} = \frac{dq_D}{dv_p} \frac{dv_p}{dv_g} = \frac{1}{n} \frac{dq_D}{dv_p}, \quad (4.22)$$

where

$$\frac{dq_D}{dv_p} = \frac{\partial q_D}{\partial q_s} \left(\frac{dv_p}{dq_s} \right)^{-1} + \frac{\partial q_D}{\partial q_d} \left(\frac{dv_p}{dq_d} \right)^{-1} = \frac{q_s}{2q_s + 1} \frac{\partial q_D}{\partial q_s} + \frac{q_d}{2q_d + 1} \frac{\partial q_D}{\partial q_d}. \quad (4.23)$$

Performing the differentiation leads to

$$\frac{dq_D}{dv_p} = \frac{1}{30} \frac{4q_s^3 + 6q_d^3 + 28q_s^2 q_d + 22q_s q_d^2 + 10q_s^2 + 15q_d^2 + 35q_s q_d + 5q_s + 10q_d}{(q_s + q_d + 1)^3} \quad (4.24)$$

and finally

$$\frac{dq_D}{dv_g} = \frac{1}{30n} \frac{4q_s^3 + 6q_d^3 + 28q_s^2 q_d + 22q_s q_d^2 + 10q_s^2 + 15q_d^2 + 35q_s q_d + 5q_s + 10q_d}{(q_s + q_d + 1)^3}. \quad (4.25)$$

4.3 Derivatives of q_S

The derivatives of q_S wrt v_d , v_s and v_g are obtained by using the source-drain symmetry, simply swapping q_s and q_d in the expressions obtained for the derivatives of q_D . This results in

$$\frac{\partial q_S}{\partial v_d} = -\frac{q_d}{30} \frac{4q_s^2 + 4q_d^2 + 12q_s q_d + 10q_s + 10q_d + 5}{(q_s + q_d + 1)^3}, \quad (4.26)$$

$$\frac{\partial q_S}{\partial v_s} = -\frac{q_s}{30} \frac{6q_s^2 + 16q_d^2 + 18q_s q_d + 15q_s + 25q_d + 10}{(q_s + q_d + 1)^3}, \quad (4.27)$$

$$\frac{dq_S}{dv_g} = \frac{1}{30n} \frac{6q_s^3 + 4q_d^3 + 22q_s^2 q_d + 28q_s q_d^2 + 15q_s^2 + 10q_d^2 + 35q_s q_d + 10q_s + 5q_d}{(q_s + q_d + 1)^3}. \quad (4.28)$$

4.4 Derivatives of q_B

The expression (3.26) can be rewritten as

$$q_B = q_{B1} + q_{B2}, \quad (4.29)$$

with

$$q_{B1} \triangleq \frac{\gamma_b}{2n} \sqrt{\psi_0 + v_p}, \quad (4.30)$$

$$q_{B2} \triangleq -\frac{n-1}{n} q_I. \quad (4.31)$$

Since q_{B1} does neither depend on v_d nor on v_s , we can write the derivatives of q_B wrt v_d and v_s as

$$\frac{\partial q_B}{\partial v_d} = \frac{\partial q_{B2}}{\partial v_d} = -\frac{n-1}{n} \frac{\partial q_I}{\partial v_d}, \quad (4.32)$$

$$\frac{\partial q_B}{\partial v_s} = \frac{\partial q_{B2}}{\partial v_s} = -\frac{n-1}{n} \frac{\partial q_I}{\partial v_s}, \quad (4.33)$$

where $\partial q_I/\partial v_d$ and $\partial q_I/\partial v_s$ are given by (4.9) and (4.10), respectively.

From (4.30) and (4.31), we see that both q_{B1} and q_{B2} depend on v_p resulting in

$$\frac{\partial q_{B1}}{\partial v_p} = \frac{\gamma_b}{4n \sqrt{\psi_0 + v_p}} = \frac{n-1}{2n}, \quad (4.34)$$

$$\frac{\partial q_{B2}}{\partial v_p} = -\frac{n-1}{n} \frac{\partial q_I}{\partial v_p}, \quad (4.35)$$

since the slope factor n is defined as

$$n = 1 + \frac{\Gamma_b}{2\sqrt{\Psi_0 + V_P}} = 1 + \frac{\gamma_b}{2\sqrt{\psi_0 + v_p}} \quad (4.36)$$

and hence

$$\frac{\gamma_b}{2\sqrt{\psi_0 + v_p}} = n - 1. \quad (4.37)$$

The derivative of q_B with respect to v_p can then be written as

$$\frac{\partial q_B}{\partial v_p} = \frac{n-1}{n} \left(\frac{1}{2} - \frac{\partial q_I}{\partial v_p} \right), \quad (4.38)$$

where $\partial q_I / \partial v_p$ is given by (4.16).

The derivative of q_B wrt v_g is finally given by

$$\frac{\partial q_B}{\partial v_g} = \frac{1}{n} \frac{\partial q_B}{\partial v_p} = \frac{n-1}{n^2} \left(\frac{1}{2} - \frac{\partial q_I}{\partial v_p} \right) \quad (4.39)$$

4.5 Derivative of q_G

Recalling that by charge conservation we have

$$q_G = -q_B - q_I. \quad (4.40)$$

Replacing q_B by (4.31) we get

$$q_G = -\frac{\gamma_b}{2n} \sqrt{\psi_0 + v_p} + \frac{n-1}{n} q_I - q_I = -\frac{1}{n} \left(\frac{\gamma_b}{2} \sqrt{\psi_0 + v_p} + q_I \right). \quad (4.41)$$

The derivative of q_G wrt v_d and v_s are simply

$$\frac{\partial q_G}{\partial v_d} = -\frac{1}{n} \frac{\partial q_I}{\partial v_d}, \quad (4.42)$$

$$\frac{\partial q_G}{\partial v_s} = -\frac{1}{n} \frac{\partial q_I}{\partial v_s}, \quad (4.43)$$

where $\partial q_I / \partial v_d$ and $\partial q_I / \partial v_s$ are given by (4.9) and (4.10), respectively.

The derivative of q_G wrt v_p is given by

$$\frac{\partial q_G}{\partial v_p} = -\frac{\partial q_B}{\partial v_p} - \frac{\partial q_I}{\partial v_p} = -\frac{1}{n} \left(\frac{n-1}{2} + \frac{\partial q_I}{\partial v_p} \right), \quad (4.44)$$

where $\partial q_I / \partial v_p$ is given by (4.16).

The derivative of q_G wrt v_g is finally given by

$$\frac{\partial q_G}{\partial v_g} = -\frac{1}{n^2} \left(\frac{n-1}{2} + \frac{\partial q_I}{\partial v_p} \right). \quad (4.45)$$

5 Intrinsic capacitances

5.1 Definition

i Note

A capacitance is controlled by the voltage across it while a transcapacitance is modelled by a VCCS controlled by the time derivative of a voltage that is not across the VCCS. In the following, we will not distinguish between capacitances and transcapacitances. We will only be talking about capacitances.

The definition of the intrinsic capacitances are given in (2.11), which is repeated below for convenience

$$C_{KK} = + \left. \frac{\partial Q_K}{\partial V_K} \right|_{op} = + \left. \frac{\partial Q_K}{\partial V_{KB}} \right|_{op}, \quad (5.1)$$

$$C_{KL} = - \left. \frac{\partial Q_K}{\partial V_L} \right|_{op} = - \left. \frac{\partial Q_L}{\partial V_{LB}} \right|_{op}, \quad (5.2)$$

with $K = D, G, S, B$ and $L = D, G, S, B$.

To keep the consistency with the results obtained by Bucher [5], we normalize the capacitance to $W L C_{ox}$, leading to

$$c_{kk} \triangleq \frac{C_{KK}}{W L C_{ox}} = -2n \left. \frac{\partial q_K}{\partial v_k} \right|_{op}, \quad (5.3)$$

$$c_{kl} \triangleq \frac{C_{KL}}{W L C_{ox}} = +2n \left. \frac{\partial q_K}{\partial v_l} \right|_{op}. \quad (5.4)$$

i Note

Notice that the signs have changed because of the original negative normalization parameter $-2n W L C_{ox} U_T$ used for the terminal charges.

i Note

We use small letters for the normalized capacitances and their subscripts to make the notation a bit lighter.

5.2 Charge conservation

From the charge conservation

$$q_G = -q_B - q_I = -q_B - q_S - q_D \quad (5.5)$$

we can write

$$\frac{\partial q_G}{\partial v_d} = -\frac{\partial q_B}{\partial v_d} - \frac{\partial q_S}{\partial v_d} - \frac{\partial q_D}{\partial v_d}, \quad (5.6)$$

$$\frac{\partial q_G}{\partial v_g} = -\frac{\partial q_B}{\partial v_g} - \frac{\partial q_S}{\partial v_g} - \frac{\partial q_D}{\partial v_g}, \quad (5.7)$$

$$\frac{\partial q_G}{\partial v_s} = -\frac{\partial q_B}{\partial v_s} - \frac{\partial q_S}{\partial v_s} - \frac{\partial q_D}{\partial v_s}, \quad (5.8)$$

$$\frac{\partial q_G}{\partial v_b} = -\frac{\partial q_B}{\partial v_b} - \frac{\partial q_S}{\partial v_b} - \frac{\partial q_D}{\partial v_b}. \quad (5.9)$$

From the definitions of the normalized capacitances this translates into

$$c_{gd} = -c_{bd} - c_{sd} + c_{dd}, \quad (5.10)$$

$$-c_{gg} = -c_{bg} - c_{sg} - c_{dg}, \quad (5.11)$$

$$c_{gs} = -c_{bs} + c_{ss} - c_{ds}, \quad (5.12)$$

$$c_{gb} = c_{bb} - c_{sb} - c_{db}. \quad (5.13)$$

or

$$c_{dd} = c_{gd} + c_{sd} + c_{bd}, \quad (5.14)$$

$$c_{gg} = c_{dg} + c_{sg} + c_{bg}, \quad (5.15)$$

$$c_{ss} = c_{ds} + c_{gs} + c_{bs}, \quad (5.16)$$

$$c_{bb} = c_{db} + c_{gb} + c_{sb} \quad (5.17)$$

5.3 Basic relations between capacitances

5.3.1 Bulk-gate “symmetry”

From (4.41), we have

$$\frac{\partial q_G}{\partial v_s} = -\frac{1}{n} \frac{\partial q_I}{\partial v_s}. \quad (5.18)$$

On the other hand from (4.33) we have

$$\frac{\partial q_B}{\partial v_s} = -\frac{n-1}{n} \frac{\partial q_I}{\partial v_s} = (n-1) \frac{\partial q_G}{\partial v_s}. \quad (5.19)$$

By symmetry we also have

$$\frac{\partial q_B}{\partial v_d} = (n-1) \frac{\partial q_G}{\partial v_d}. \quad (5.20)$$

This leads to the following relations between capacitances

$$c_{bd} = (n-1) c_{gd}, \quad (5.21)$$

$$c_{bs} = (n-1) c_{gs}. \quad (5.22)$$

Similarly we also have

$$c_{db} = (n-1) c_{dg}, \quad (5.23)$$

$$c_{sb} = (n-1) c_{sg}. \quad (5.24)$$

The derivative of q_B with respect to v_p can be written as

$$\frac{\partial q_B}{\partial v_p} = \frac{n-1}{n} \left(\frac{1}{2} - \frac{\partial q_I}{\partial v_p} \right) = \frac{n-1}{n} \left(\frac{1}{2} - \frac{\partial q_D}{\partial v_p} - \frac{\partial q_S}{\partial v_p} \right) \quad (5.25)$$

This translates into a relation between c_{bg} , c_{dg} and c_{sg}

$$c_{bg} = \frac{n-1}{n} (1 - c_{dg} - c_{sg}). \quad (5.26)$$

From (3.30), it is easy to show that

$$c_{gg} = c_{bg} + c_{sg} + c_{dg}. \quad (5.27)$$

Replacing c_{bg} by (5.26) results in

$$c_{gg} = \frac{1}{n} (n-1 + c_{dg} + c_{sg}). \quad (5.28)$$

We also can show that

$$c_{dg} = \frac{c_{dd} - c_{ds}}{n}, \quad (5.29)$$

$$c_{sg} = \frac{c_{ss} - c_{sd}}{n}. \quad (5.30)$$

We finally have the following relations

$$c_{bd} = (n-1) c_{gd}, \quad (5.31)$$

$$c_{bs} = (n-1) c_{gs}, \quad (5.32)$$

$$c_{db} = (n-1) c_{dg}, \quad (5.33)$$

$$c_{sb} = (n-1) c_{sg}, \quad (5.34)$$

$$c_{bg} = \frac{n-1}{n} (1 - c_{dg} - c_{sg}), \quad (5.35)$$

$$c_{gb} = c_{bg}, \quad (5.36)$$

$$c_{gg} = \frac{1}{n} (n-1 + c_{dg} + c_{sg}), \quad (5.37)$$

$$c_{dg} = \frac{c_{dd} - c_{ds}}{n}, \quad (5.38)$$

$$c_{sg} = \frac{c_{ss} - c_{sd}}{n}, \quad (5.39)$$

$$c_{bb} = c_{db} + c_{gb} + c_{sb}. \quad (5.40)$$

5.4 Primary intrinsic capacitances

We can choose the following 6 capacitances c_{gs} , c_{ds} , c_{ss} , c_{gd} , c_{sd} and c_{dd} as primary capacitances and express the other 10 capacitances in terms of these 6 capacitances using (5.31) to (5.40). Additionally, we can use the source-drain symmetry to express c_{gd} in terms of c_{gs} , c_{sd} in terms of c_{ss} and c_{dd} in terms of c_{ds} by swapping q_s and q_d . We therefore need only to evaluate c_{gs} , c_{ds} and c_{ss} which are given below

$$c_{gs}(q_s, q_d) = \frac{q_s}{3} \frac{4q_d + 2q_s + 3}{(q_s + q_d + 1)^2}, \quad (5.41)$$

$$c_{ds}(q_s, q_d) = -\frac{n}{15} q_s \frac{4q_s^2 + 4q_d^2 + 12q_s q_d + 10q_s + 10q_d + 5}{(q_s + q_d + 1)^3}, \quad (5.42)$$

$$c_{ss}(q_s, q_d) = \frac{n}{15} q_s \frac{6q_s^2 + 16q_d^2 + 18q_s q_d + 15q_s + 25q_d + 10}{(q_s + q_d + 1)^3}. \quad (5.43)$$

c_{gd} , c_{sd} and c_{dd} are then given by

$$c_{gd}(q_s, q_d) = c_{gs}(q_d, q_s), \quad (5.44)$$

$$c_{sd}(q_s, q_d) = c_{ds}(q_d, q_s), \quad (5.45)$$

$$c_{dd}(q_s, q_d) = c_{ss}(q_d, q_s). \quad (5.46)$$

We can then complete the capacitances related to the drain and source terminal charges as follows

$$c_{dg} = \frac{c_{dd} - c_{ds}}{n}, \quad (5.47)$$

$$c_{db} = (n - 1) c_{dg}, \quad (5.48)$$

$$c_{sg} = \frac{c_{ss} - c_{sd}}{n}, \quad (5.49)$$

$$c_{sb} = (n - 1) c_{sg}. \quad (5.50)$$

Note that, since c_{dd} , c_{ds} , c_{ss} and c_{sd} are proportional to n , c_{dg} and c_{sg} don't depend on n .

We can then complete evaluating the remaining capacitances related to the gate and bulk terminal charges as

$$c_{gg} = \frac{n - 1 + c_{gs} + c_{gd}}{n}, \quad (5.51)$$

$$c_{bg} = \frac{n - 1}{n} (1 - c_{dg} - c_{sg}), \quad (5.52)$$

$$c_{gb} = c_{bg}, \quad (5.53)$$

$$c_{bd} = (n - 1) c_{gd}, \quad (5.54)$$

$$c_{bs} = (n - 1) c_{gs}, \quad (5.55)$$

$$c_{bb} = c_{db} + c_{gb} + c_{sb}. \quad (5.56)$$

All the normalized capacitances are plotted below versus v_p and v_d .

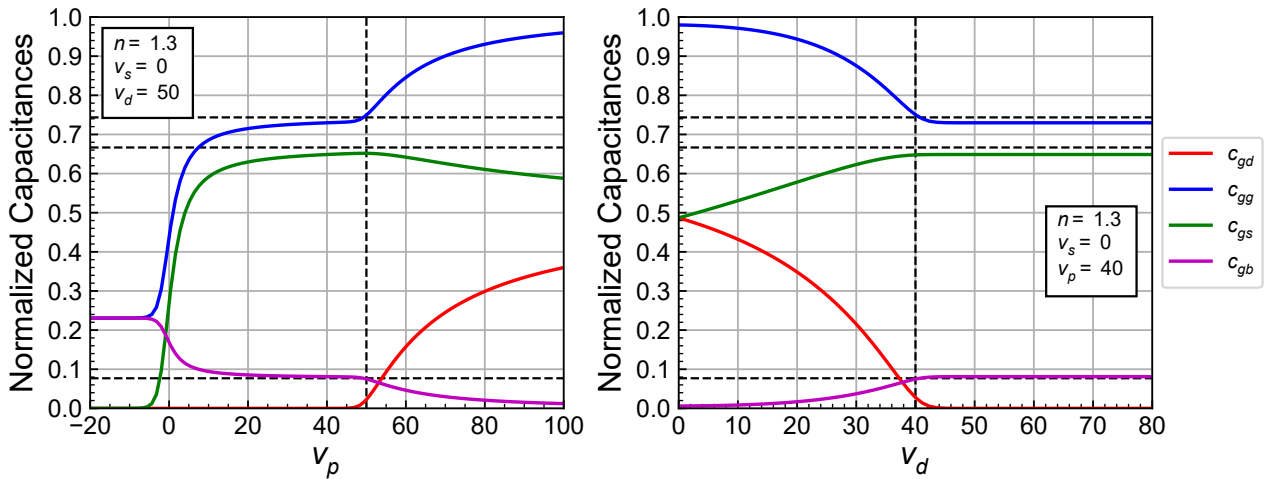


Figure 5.1: Normalized intrinsic capacitances related to the gate terminal charge plotted versus v_p and v_d .

The left plot of Figure 5.1 shows the four capacitances related to the gate terminal charge versus v_p for $v_d = 50$. c_{gd} remains zero in saturation and starts to increase when leaving saturation for $v_p > v_d$. c_{gg} starts at $(n - 1)/n = 0.23$ and then increases to $2/3 + (n - 1)/(3n) = 0.74$ in saturation and finally tends to one when leaving saturation. c_{gs} starts at zero and increases to $2/3$ in strong inversion and saturation and then decreases when leaving saturation for $v_p > v_d$ to ultimately tend to 0.5. Finally,

c_{gb} starts at $(n-1)/n = 0.23$ and then decreases to $(n-1)/(3n) = 0.08$ in saturation and finally decreases to zero.

The right plot of Figure 5.1 shows the same capacitances versus v_d for $v_p = 40$. c_{gd} starts at 0.5 and tends to zero in saturation for $v_d > v_p = 40$. c_{gg} starts at 1 and decreases to $2/3 + (n-1)/(3n) = 0.74$ in saturation. c_{gs} starts at 0.5 and increases to reach $2/3$ in saturation. Finally c_{gb} starts at zero and increases to reach $(n-1)/(3n) = 0.08$ in saturation.

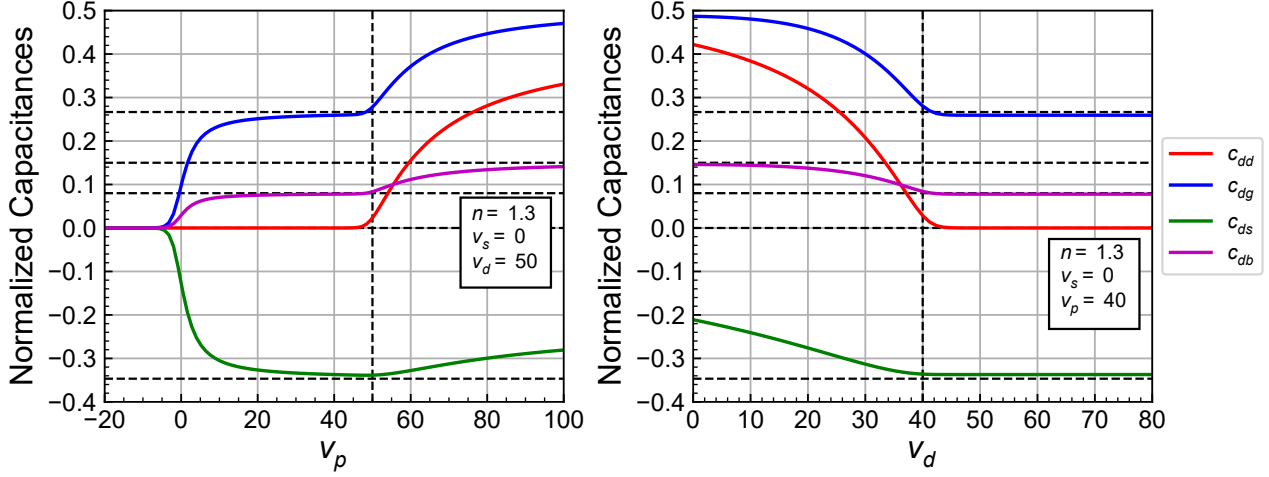


Figure 5.2: Normalized intrinsic capacitances related to the drain terminal charge plotted versus v_p and v_d .

The left plot of Figure 5.2 shows the four capacitances related to the drain terminal charge versus v_p for $v_d = 50$. c_{dd} remains zero in saturation and then increases when leaving saturation for $v_p > v_d = 50$. c_{dg} starts at zero and reaches $4/15 = 0.27$ in strong inversion and saturation and finally tends to 0.5 when leaving saturation. c_{ds} starts at zero and becomes equal to $-4n/15 = -0.35$ in strong inversion and saturation. c_{db} starts at zero and then reaches $4(n-1)/15 = 0.08$ in strong inversion and saturation and then tends to $(n-1)/2 = 0.15$ when leaving saturation.

The right plot of Figure 5.2 shows the same four capacitances versus v_d for $v_p = 40$. c_{dd} starts at $n/3 = 0.43$ and tends to zero in saturation. c_{dg} starts at 0.5 and tends to $4/15 = 0.27$ in saturation. c_{ds} starts at $-n/6 = -0.22$ and tends to $-4n/15 = -0.35$ in saturation. Finally, c_{db} starts at $(n-1)/2 = 0.15$ and tends to $4(n-1)/15 = 0.08$ in saturation.

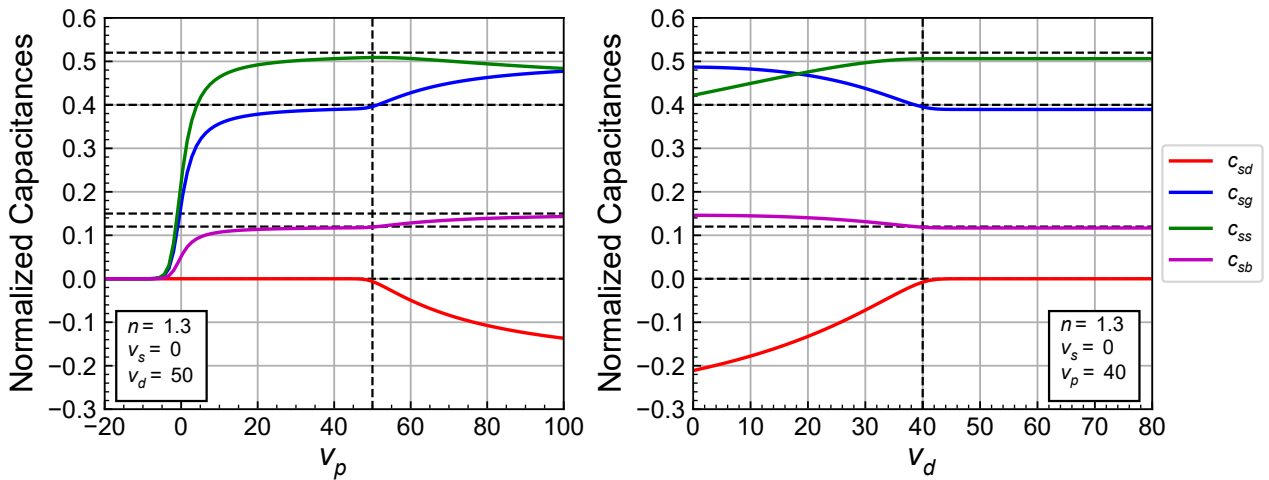


Figure 5.3: Normalized intrinsic capacitances related to the source terminal charge plotted versus v_p and v_d .

The left plot of Figure 5.3 shows the four capacitances related to the drain terminal charge versus v_p for $v_d = 50$. c_{sd} remains zero in strong inversion and saturation and then tends to $-n/6 = -0.22$ when leaving saturation. c_{sg} starts at zero and reaches $2/5 = 0.40$ in strong inversion and saturation. c_{ss} starts at zero and reaches $2n/5 = 0.52$ in strong inversion and saturation. c_{sb} starts at zero and then reaches $2(n-1)/5 = 0.12$ in strong inversion and saturation and then tends to $(n-1)/2 = 0.15$ when leaving saturation.

The right plot of Figure 5.3 shows the same four capacitances versus v_d for $v_p = 40$. c_{sd} starts at $-n/6 = -0.22$ and tends to zero in saturation. c_{sg} starts at 0.5 and tends to $2/5 = 0.40$ in saturation. c_{ss} starts at $n/3 = 0.43$ and tends to $2n/5 = 0.52$ in saturation. Finally, c_{sb} starts at $(n-1)/2 = 0.15$ and tends to $2(n-1)/5 = 0.12$ in saturation.

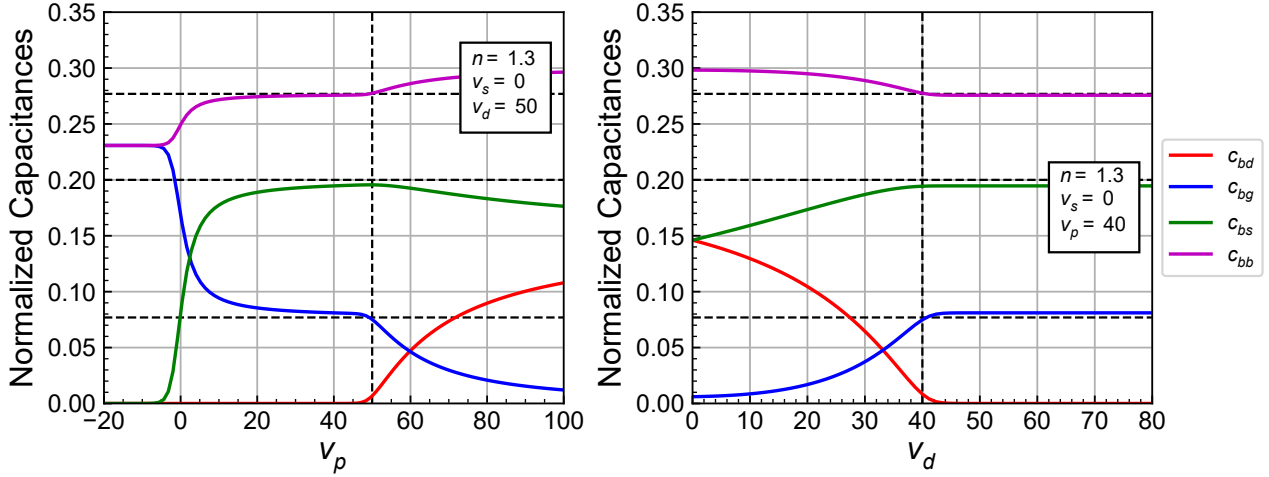


Figure 5.4: Normalized intrinsic capacitances related to the bulk terminal charge plotted versus v_p and v_d .

The left plot of Figure 5.4 shows the four capacitances related to the bulk terminal charge versus v_p for $v_d = 50$. c_{bd} remains zero in saturation and increases when leaving saturation to tend to $(n-1)/2 = 0.15$. c_{bg} starts at $(n-1)/n = 0.23$ and decreases to $(n-1)/(3n) = 0.08$ in saturation to finally decrease to zero when leaving saturation. c_{bs} starts at zero and increases to $2(n-1)/3 = 0.20$ in saturation to decrease to reach $(n-1)/n = 0.23$. Finally, c_{bb} starts at $(n-1)/n =$ and reaches $(2n-1/n-1)/3 = 0.28$ in strong inversion and saturation and then tends to $n-1 = 0.30$ when leaving saturation.

The right plot of Figure 5.4 shows the same four capacitances versus v_d for $v_p = 40$. c_{bd} starts at $(n-1)/2 = 0.15$ and decreases to reach zero in saturation. c_{bg} starts at zero and increases to $(n-1)/(3n) = 0.08$ in saturation. c_{bs} starts at $(n-1)/n = 0.23$ and increases to $2(n-1)/3 = 0.20$ in saturation. Finally, c_{bb} starts at $n-1 = 0.30$ and decreases to $(2n-1/n-1)/3 = 0.28$ in saturation.

Having all the capacitances, we now will map them into a symmetrical and convenient equivalent small-signal circuit.

6 Small-signal equivalent circuit

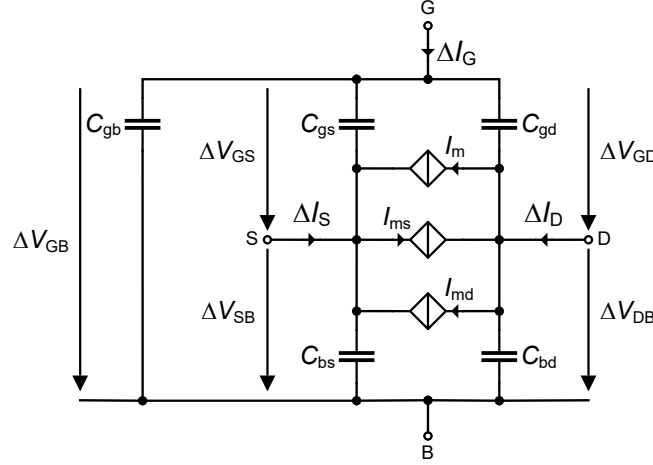


Figure 6.1: Equivalent small-signal circuit for the charging currents [6] [7] [1]

There are many equivalent circuits that can implement the terminal charging currents $\Delta I_{D,ac}$, ΔI_G , $\Delta I_{S,ac}$ and ΔI_B . In the case of the EKV model, we want to map the charging currents to the small-signal circuit shown in Figure 6.1 [6] [7] [1], where

$$I_m = -C_m \frac{dV_{GB}}{dt}, \quad (6.1)$$

$$I_{ms} = -C_{ms} \frac{dV_{SB}}{dt}, \quad (6.2)$$

$$I_{md} = -C_{md} \frac{dV_{DB}}{dt}. \quad (6.3)$$

The terminal charging currents are then given by

$$\Delta I_{D,ac}(t) = -C_{md} \frac{dV_{DB}}{dt} - C_m \frac{dV_{GB}}{dt} + C_{ms} \frac{dV_{SB}}{dt} - C_{gd} \frac{dV_{GD}}{dt} + C_{bd} \frac{dV_{DB}}{dt}, \quad (6.4)$$

$$\Delta I_G(t) = C_{gb} \frac{dV_{GB}}{dt} + C_{gs} \frac{dV_{GS}}{dt} + C_{gd} \frac{dV_{GD}}{dt}, \quad (6.5)$$

$$\Delta I_{S,ac}(t) = C_{md} \frac{dV_{DB}}{dt} + C_m \frac{dV_{GB}}{dt} - C_{ms} \frac{dV_{SB}}{dt} - C_{gs} \frac{dV_{GS}}{dt} + C_{bs} \frac{dV_{SB}}{dt}. \quad (6.6)$$

Now

$$V_{GS} = V_{GB} - V_{SB}, \quad (6.7)$$

$$V_{GD} = V_{GB} - V_{DB}. \quad (6.8)$$

Replacing in the above equations results in

$$\Delta I_{D,ac}(t) = (C_{bd} + C_{gd} - C_{md}) \frac{dV_{DB}}{dt} - (C_m + C_{gd}) \frac{dV_{GB}}{dt} + C_{ms} \frac{dV_{SB}}{dt}, \quad (6.9)$$

$$\Delta I_G(t) = -C_{gd} \frac{dV_{DB}}{dt} + (C_{gb} + C_{gd} + C_{gs}) \frac{dV_{GB}}{dt} - C_{gs} \frac{dV_{SB}}{dt}, \quad (6.10)$$

$$\Delta I_{S,ac}(t) = C_{md} \frac{dV_{DB}}{dt} + (C_m - C_{gs}) \frac{dV_{GB}}{dt} + (C_{bs} + C_{gs} - C_{ms}) \frac{dV_{SB}}{dt}. \quad (6.11)$$

In order for the equivalent circuit to produce the same terminal charging currents for the same changes in the terminal voltages, the coefficient of the terminal voltage derivatives must be equal. Equating the coefficients of the drain, gate and source voltage derivatives and solving for C_m , C_{ms} and C_{md} results in

$$C_m = C_{dg} - C_{gd}, \quad (6.12)$$

$$C_{ms} = -C_{ds}, \quad (6.13)$$

$$C_{md} = -C_{sd}. \quad (6.14)$$

Notice that the transcapacitances C_{ms} and C_{md} are actually positive since C_{ds} and C_{sd} are negative.

Similarly for the source terminal current, we get

$$C_m = C_{gs} - C_{sg}, \quad (6.15)$$

$$C_{ms} = -C_{ds}, \quad (6.16)$$

$$C_{md} = -C_{sd}. \quad (6.17)$$

The values found for C_{ms} and C_{md} are identical. However the values for C_m are different. The equality between the two expressions of C_m

$$C_m = C_{dg} - C_{gd} = C_{gs} - C_{sg} \quad (6.18)$$

only holds if $C_{gb} = C_{bg}$ which is actually the case. We can therefore state that the small-signal circuit shown in Figure 6.1 produces the correct terminal charging currents.

We can now add the transconductances and embed them into the VCCS, giving

$$I_m = Y_m \cdot \Delta V_{GB}, \quad (6.19)$$

$$I_{ms} = Y_{ms} \cdot \Delta V_{SB}, \quad (6.20)$$

$$I_{md} = Y_{md} \cdot \Delta V_{DB}, \quad (6.21)$$

with [1]

$$Y_m = G_m - s C_m = G_m (1 - s \tau_{qs}), \quad (6.22)$$

$$Y_{ms} = G_{ms} - s C_{ms} = G_{ms} (1 - s \tau_{qs}), \quad (6.23)$$

$$Y_{md} = G_{md} - s C_{md} = G_{md} (1 - s \tau_{qs}). \quad (6.24)$$

τ_{qs} is the quasi-static time constant which is identical for Y_m , Y_{ms} and Y_{md} and given by [1]

$$\tau_{qs} = \frac{C_m}{G_m} = \frac{C_{ms}}{G_{ms}} = \frac{C_{md}}{G_{md}}. \quad (6.25)$$

The normalized quasi-static time constant can be expressed in terms of q_s and q_d as [1]

$$\frac{\tau_{qs}}{\tau_{spec}} = \frac{\omega_{spec}}{\omega_{qs}} = \frac{1}{30} \frac{4q_s^2 + 4q_d^2 + 12q_s q_d + 10q_s + 10q_d + 5}{(q_s + q_d + 1)^3}, \quad (6.26)$$

where

$$\tau_{spec} = \frac{1}{\omega_{spec}} = \frac{L^2}{\mu U_T}. \quad (6.27)$$

ω_{qs} gives an upper frequency limit for the validity of the QS model. Beyond this limit, the channel behaves more like a non-uniform distributed transmission line and the lumped model of Figure 6.1 is no more valid. A non-quasi-static (NQS) model is then required [6] [7] [1].

The five intrinsic capacitances of the small-signal circuit of Figure 6.1 are plotted in Figure 6.2 and the three transcapacitances are plotted versus v_p and v_d in Figure 6.4.

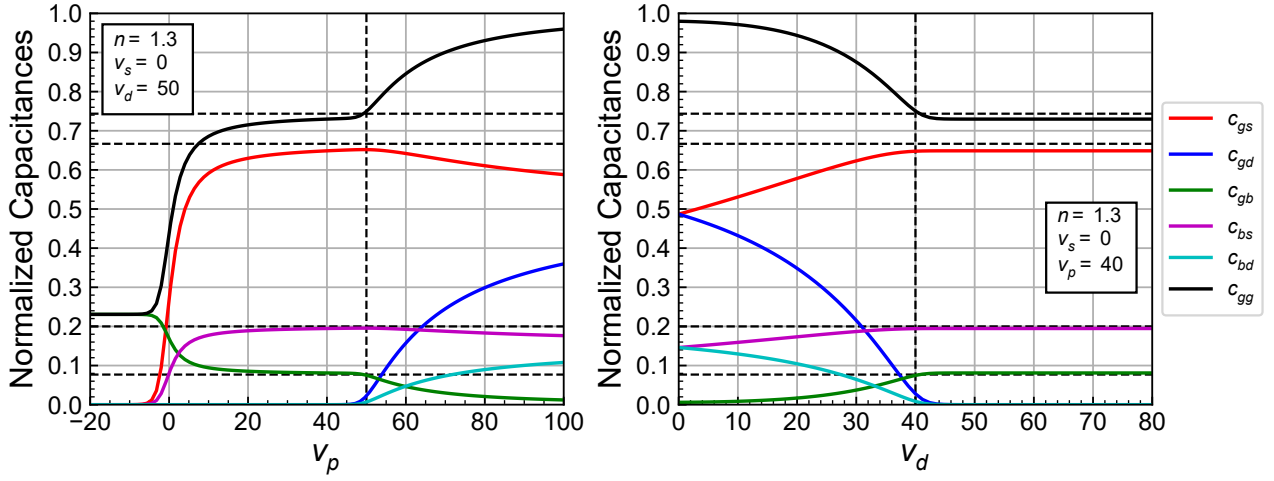


Figure 6.2: Normalized intrinsic capacitances of the equivalent small-signal circuit of Figure 6.1 plotted versus v_p and v_d .

The plots shown in Figure 6.2 are calculated with a constant slope factor n . This has no impact on c_{gs} and c_{gd} since they don't depend on n . However, c_{gb} , c_{bs} and c_{bd} depend n and are therefore affected by the variation of n with v_p . This is illustrated in Figure 6.3 where the dashed lines have been calculated accounting for the dependence of n to v_p . We see some differences mainly in c_{gb} , c_{bs} , c_{bd} and c_{gg} . The differences remain small and the approximation of considering n as constant is sufficient for most purposes.

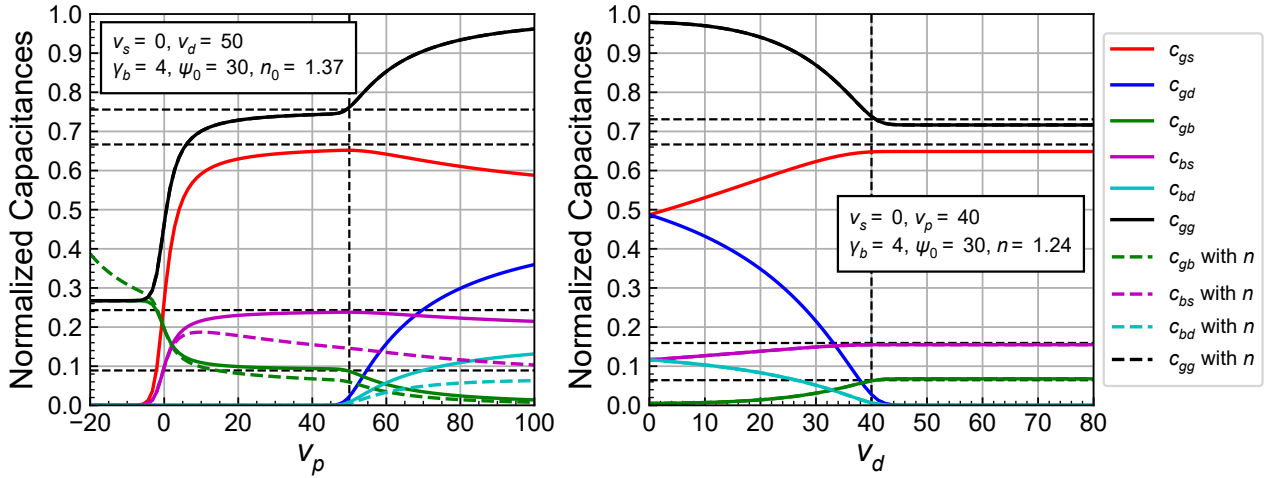


Figure 6.3: Normalized intrinsic capacitances of the equivalent small-signal circuit of Figure 6.1 plotted versus v_p and v_d including the dependence of n with v_p .

Similarly to the capacitances, the transcapacitances are affected by a change of n with v_p . Actually, since c_{gs} and c_{sg} don't depend on n , c_m does not depend on n neither. However, c_{ms} and c_{md} do depend on n as shown in Figure 6.5.

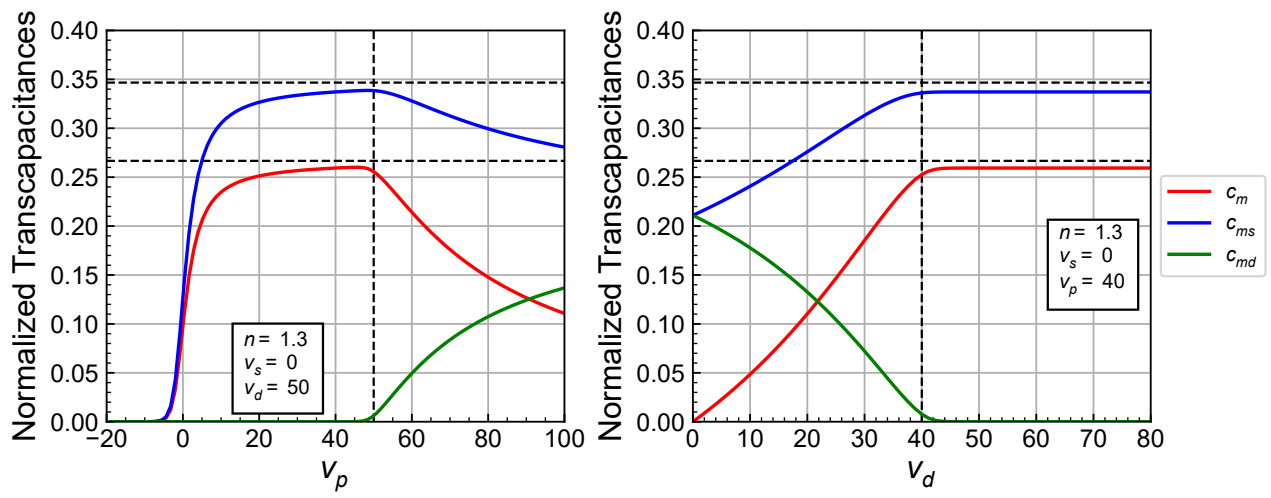


Figure 6.4: Normalized intrinsic transcapacitances of the equivalent small-signal circuit of Figure 6.1 plotted versus v_p and v_d .

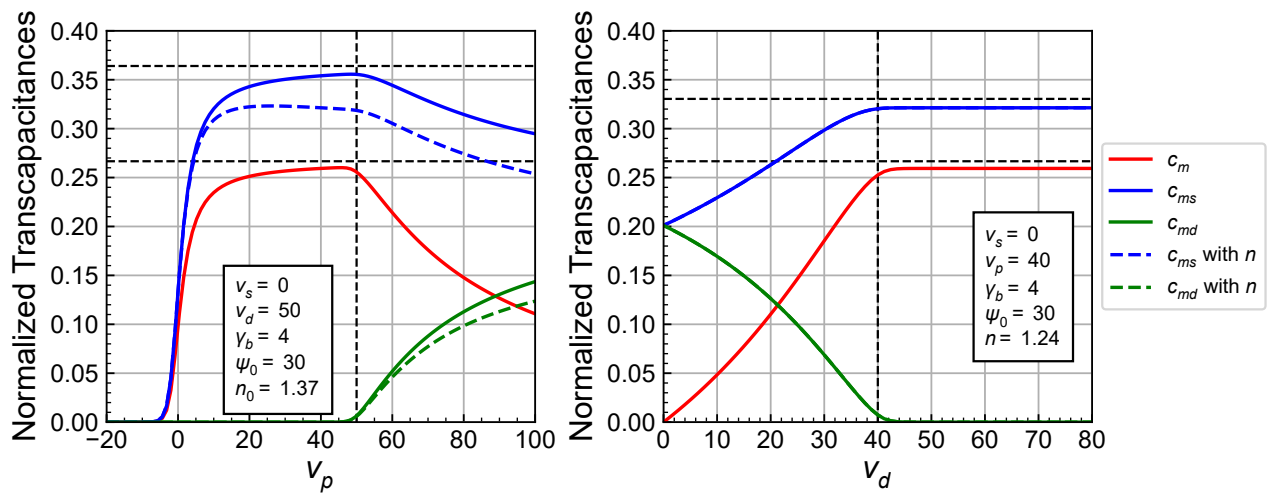


Figure 6.5: Normalized intrinsic transcapacitances of the equivalent small-signal circuit of Figure 6.1 plotted versus v_p and v_d including the dependence of n with v_p .

7 Conclusion

In this notebook we have derived the high-frequency quasi-static (QS) small-signal model of the intrinsic part of the MOS transistor. We have started by separating the small-signal drain current between a DC part due to the transconductances and an AC part that is due to the charging currents. We have identified the 16 capacitances required to describe the AC small-signal charging currents and shown that they are not all independent and that we actually only need 9 independent capacitances.

The capacitances can be evaluated from the terminal charges. We therefore have derived the four terminal charges under QS assumption. The normalized source and drain terminal charges and the total inversion charge in the channel only depend on q_s and q_d , while the gate and bulk terminal charges depend on the total inversion charge and on the pinch-off voltage through the parameters γ_b and ψ_0 related to the depletion capacitance.

We then have calculated all the derivatives of the terminal charges wrt to the controlling voltages that are required for deriving the capacitances assuming that the slope factor n remains constant. We have then expressed the derivatives of the normalized terminal charges in terms of n , q_s and q_d .

We have then used these derivatives to express the normalized intrinsic capacitances in terms of n , q_s and q_d . We additionally have derived 10 relations between the various intrinsic capacitances and shown that we only need to know three primary intrinsic capacitances, three others can then be expressed by using the source-drain symmetry by swapping q_s and q_d and the other ten can then be expressed by using the 10 capacitance relations.

Finally, we have shown that the charging currents can be mapped into an equivalent small-signal circuit including 5 capacitances and 3 transcapacitances modelled by 3 VCCS which can then be combined with the 3 transconductances to obtain the final complete QS small-signal circuit of the intrinsic part of the transistor.

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