

# Noise Sampling

Version 1

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This notebook investigates the effect of sampling on noise. It starts looking at the ideal sample-and-hold (S&H) case in which the signal is acquired almost instantaneously and then held for the rest of the period. It then looks at the track-and-hold (T&H) case and investigates how the noise is affected by the duty cycle. A simple approximation of the output power spectral density is obtained for both the S&H and the T&H for an input noise including white and flicker noise.

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# Licensing

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# 1 Introduction

In this notebook we investigate the effect of sampling on noise. We first look at the sample-and-hold case in which the signal is acquired almost instantaneously and then held for the rest of the period. Then we will look at the track-and-hold case and investigate how the noise is affected by the duty cycle. Many of the theoretical results are based on [1].

## 2 The simple track-and-hold circuit

The simplest track-and-hold circuit is shown in Fig. Figure 2.1 and the corresponding waveforms are illustrated in Fig. Figure 2.2.

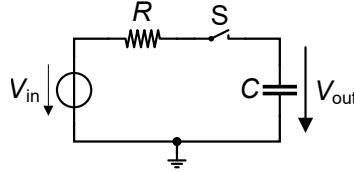


Figure 2.1: The basic track-and-hold circuit.

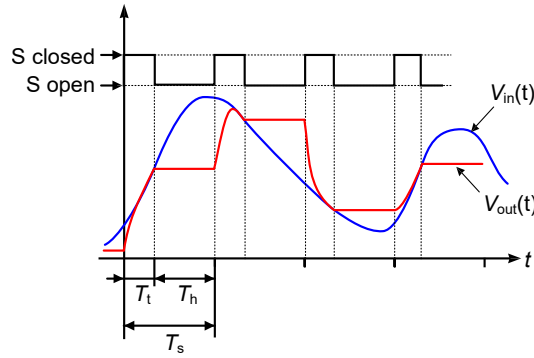


Figure 2.2: Track-and-hold waveforms.

During the track phase, the switch is closed and the output voltage on the capacitor is catching up the input voltage with a certain time constant  $\tau \triangleq RC$  where  $R$  represents the combined resistances of the switch and of the source. Once the output signal follows the input signal, which is assumed to have a bandwidth much smaller than the  $RC$  cut-off frequency  $f_c = 1/(2\pi RC)$ , at the end of the track phase the switch is opened and the input voltage at time  $nT_s + T_t$  is sampled on the hold capacitor  $C$  and held until the next period.

In the case the  $RC$  time constant is much smaller than the track time  $T_t$ , the acquisition of the signal can be done almost instantaneously and the sampled voltage is then held for almost the whole sampling period  $T_s$ . The waveform at the output is then a simple staircase with values corresponding to the values of the signal at times  $nT_s$ . This particular case is called ideal sample-and-hold and will be analyzed in the next Section.

## 3 Sample-and-hold

### 3.1 The ideal sample-and-hold model

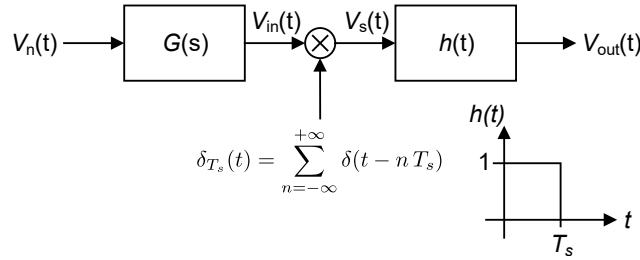


Figure 3.1: Block diagram model of the ideal sample-and-hold circuit.

The ideal sample-and-hold (S&H) circuit discussed above can be modeled by the block diagram shown in Fig. Figure 3.1 where the first block represents a low-pass filter accounting for the low-pass characteristic introduced by the RC network and having a transfer function given by

$$G(s) = \frac{1}{1 + s \cdot \tau} \quad (3.1)$$

with  $\tau = RC$ . The input voltage  $V_{in}(t)$  corresponding to the low-pass filtered noise signal  $V_n(t)$  is then sampled giving the sampled voltage  $V_s$

$$V_s(t) = V_{in}(t) \cdot \delta_{T_s}(t) = V_{in}(t) \cdot \sum_{n=-\infty}^{+\infty} \delta(t - n T_s) = \sum_{n=-\infty}^{+\infty} V_{in}(n T_s) \delta(t - n T_s). \quad (3.2)$$

The ideal sampled voltage  $V_s(t)$  is then convoluted with the filter having a rectangular impulse response  $h(t)$

$$h(t) = \begin{cases} 1 & \text{for } 0 < t \leq T_s, \\ 0 & \text{otherwise.} \end{cases} \quad (3.3)$$

The sample-and-hold output signal is then given by

$$V_{out}(t) = h(t) * \sum_{n=-\infty}^{+\infty} V_{in}(n T_s) \delta(t - n T_s) = \sum_{n=-\infty}^{+\infty} V_{in}(n T_s) h(t - n T_s), \quad (3.4)$$

where the  $*$  symbol corresponds to the convolution operator.

In the case the input signal is a stationary random process in the wide-sense [2], the sample-and-hold output signal is a sequence of constant steps during each period with an amplitude varying randomly from one period to the other. It can be shown that the sampled signal can no more be considered as stationary [3] [4] [5] [6] (check the link to Ken Kundert's web The [Designer's Guide](#) where you will find many useful information about this topic). However, It can be shown that if the signal is not observed in synchronism with the clock period, the non-stationary process can be treated as if it was stationary by simply time averaging the mean value and the autocorrelation function over one period. This procedure is important because the concepts used for stationary processes, and particularly the Power Spectral Density (PSD) can be valid for evaluating such a system performance. This is the

case in most circuits like Switched-Capacitor (SC) circuits where the noise is measured over a large number of clock periods. Therefore the S&H output signal will subsequently be treated as if it was stationary.

It can be shown that the PSD (actually the PSD averaged over one period) is given by [1]

$$S_{out}(f) = \text{sinc}^2(\pi f T_s) \cdot \sum_{n=-\infty}^{+\infty} S_{in} \left( f - \frac{n}{T_s} \right), \quad (3.5)$$

where

$$\text{sinc}(\pi f T_s) \triangleq \frac{\sin(\pi f T_s)}{\pi f T_s} \quad (3.6)$$

is the *sinus cardinal* function. The PSD of the sampled-and-held output signal is then simply the superposition of the input PSD shifted to the multiples of the sampling frequency and multiplied by the  $\text{sinc}^2(\pi f T_s)$  due to the hold function. Since the bandwidth of the S&H circuit needs to be larger than the signal bandwidth in order for the signal to be acquired within the track period  $T_t$ , the broadband noise will be aliased and folded back to the Nyquist band.

In the next section, we will look at the sampling of a white input noise.

## 3.2 White noise sampling

### 3.2.1 Theory

The effect of noise folding can be illustrated in the case the input signal is an ideally low-pass filtered white noise having a PSD  $S_0$  over a bandwidth  $B_n$  equal to twice the sampling frequency or four times the Nyquist frequency as shown in Fig. Figure 3.2.

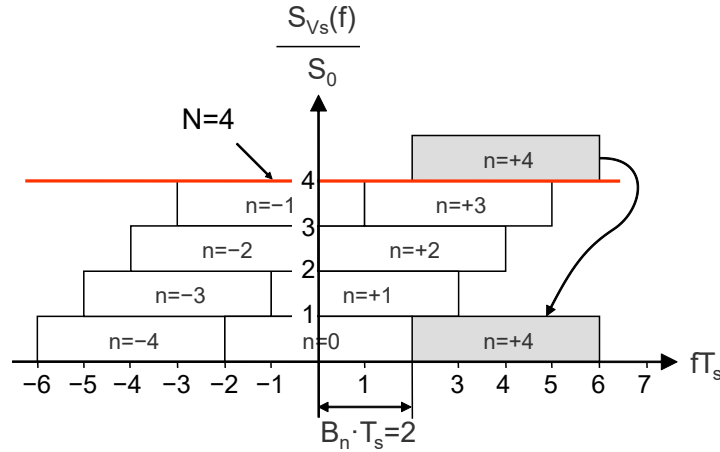


Figure 3.2: Sampling of an ideally low-pass filtered white noise having a bandwidth  $B_n \cdot T_s = 4$ .

In this example, we see that the PSD of the ideally sampled signal  $V_s(t)$  is actually a white noise equal to  $N_{under}$  times the PSD of the input signal  $S_0$

$$S_{V_s}(f) = \sum_{n=-\infty}^{+\infty} S_{in} \left( f - \frac{n}{T_s} \right) = N_{under} \cdot S_0, \quad (3.7)$$

where  $N_{under}$  is the *undersampling factor* defined as the ratio of the equivalent noise bandwidth  $B_n$  and the Nyquist frequency  $f_s/2$  [1]

$$N_{under} = 2B_n T_s = \frac{B_n}{f_s/2}. \quad (3.8)$$

This means that the high frequency noise is folded back to the Nyquist band resulting in an increase of the PSD by the undersampling factor [1]. The signal  $V_s(t)$  has no physical reality, because it has an infinite power. The power is actually bounded by the  $\text{sinc}^2(\pi f T_s)$  function introduced by the hold operation leading to the PSD of the sample-and-held output signal

$$S_{out}(f) = N_{under} \cdot S_0 \cdot \text{sinc}^2(\pi f T_s). \quad (3.9)$$

We will now derive the PSD of the sample-and-held signal in the more realistic case of a 1<sup>st</sup>-order low-pass filtered white noise having a PSD

$$S_{in}(f) = \frac{S_0}{1 + (f/f_c)^2}. \quad (3.10)$$

The shifted versions of the input PSD are plotted in Figure 3.3 for  $f_c T_s = 0.5$ .

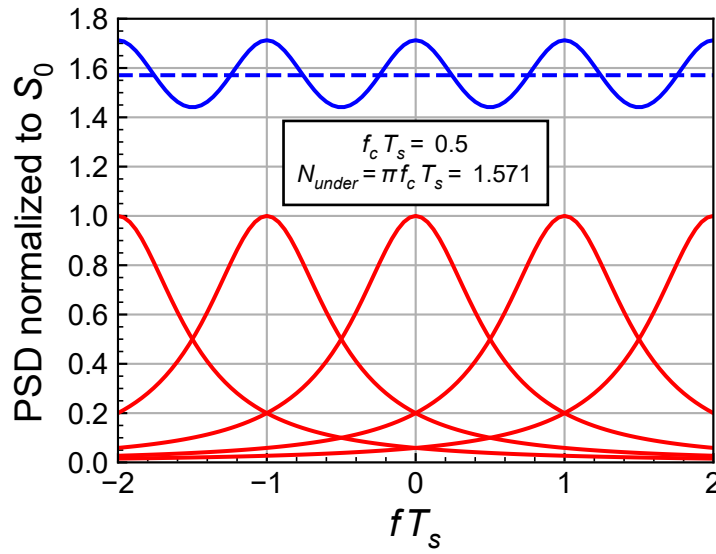


Figure 3.3: Shifted versions of the 1<sup>st</sup>-order LP filtered white noise and resulting sampled PSD.

It can be shown that the PSD of the ideal sampled signal can be calculated analytically (for example in Mathematica) as [7]

$$S_{V_s}(f) = \sum_{n=-\infty}^{+\infty} S_{in}\left(f - \frac{n}{T_s}\right) = \pi f_c T_s \cdot S_0 \cdot G(f, f_c, T_s) \quad (3.11)$$

where [7]

$$G(f, f_c, T_s) \triangleq \frac{\sinh(2\pi f_c T_s)}{\cosh(2\pi f_c T_s) - \cos(2\pi f T_s)}. \quad (3.12)$$

The PSD  $S_{V_s}$  is plotted in Figure 3.3 in blue. We see that summing the shifted spectra gives rise to a PSD that oscillates around the undersampling factor  $N_{under} = \pi f_c T_s = 1.571$ . This ripple is captured by the function  $G(f, f_c, T_s)$  which is plotted below for various cut-off frequencies.

We see from Figure 3.4 that the ripple quickly vanishes as the cut-off frequency gets closer to the sampling frequency. The effect of sampling a 1<sup>st</sup>-order low-pass filtered white noise is illustrated in Figure 3.5 for different cut-off frequencies. The theoretical expression given by (3.11) is compared to the numerical evaluation calculated with  $N$  terms in the summation.

We see that the ripple basically disappears as the cut-off frequency gets larger than the sampling frequency and  $G(f = 0, f_c, T_s)$  tends to 1 as shown in Figure 3.6.

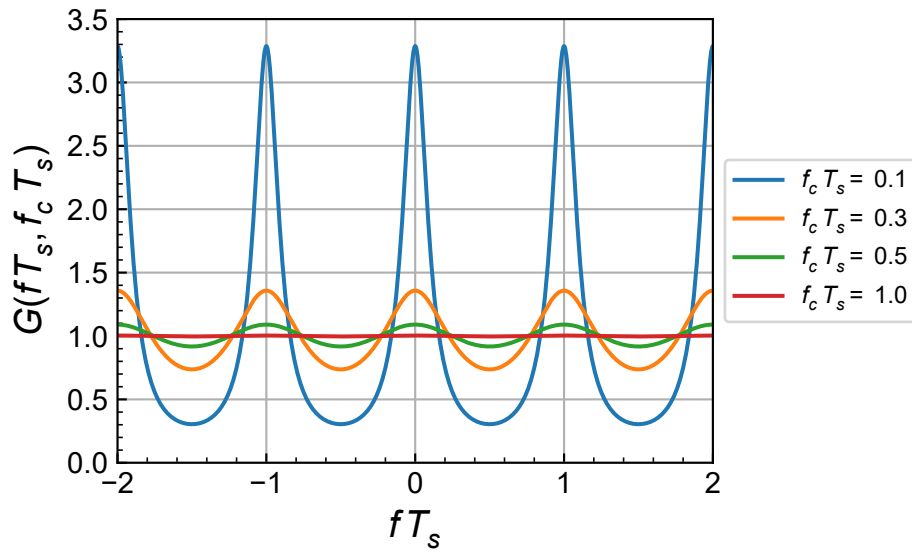


Figure 3.4: Function  $G(f, f_c, T_s)$  versus  $f T_s$  for different values of  $f_c T_s$ .

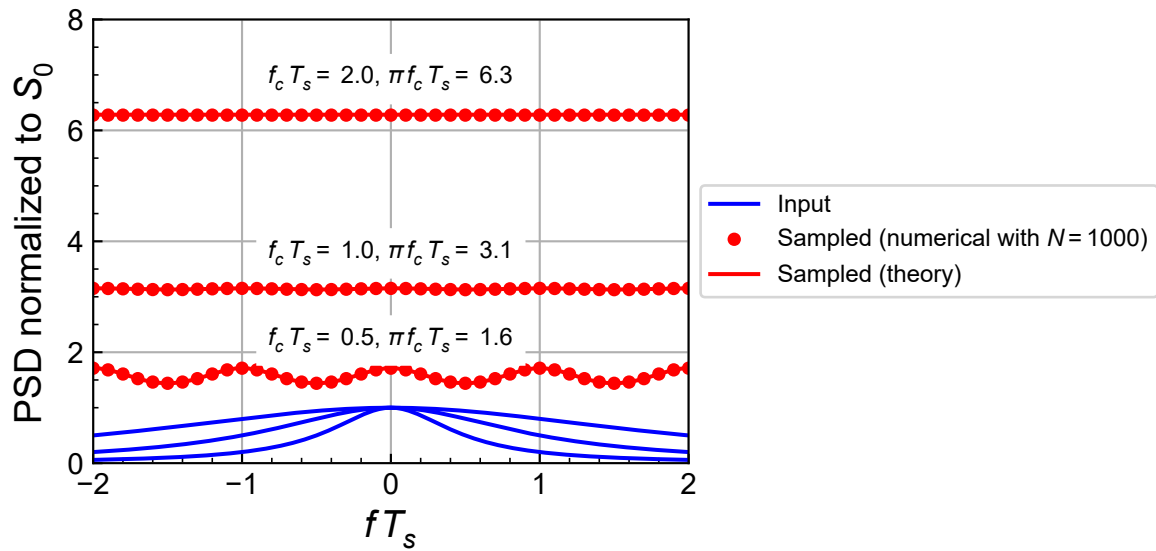


Figure 3.5: Sampled white noise PSD.

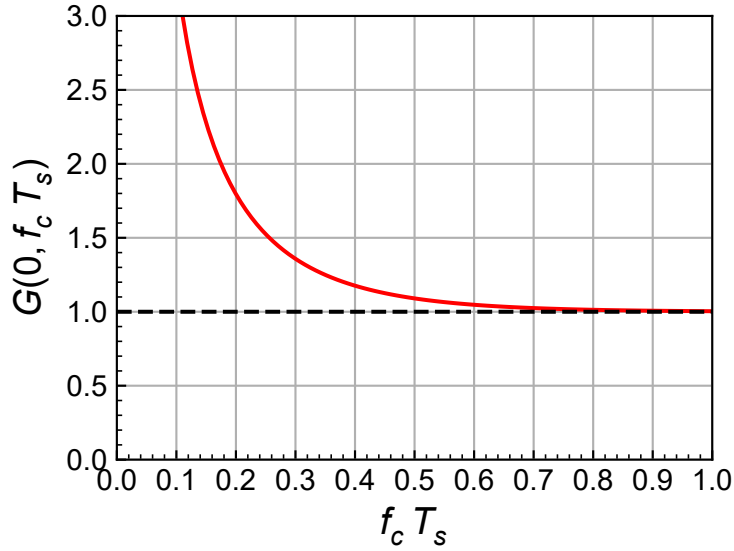


Figure 3.6: Function  $G(f = 0, f_c, T_s)$  versus  $f_c T_s$ .

Since in the real S&H circuit, the cut-off frequency needs to be larger than the sampling frequency in order for the signal to settle quickly before being sampled, we can simply replace  $G(f, f_c, T_s)$  by one, leading to

$$S_{V_s}(f) = \pi f_c T_s \cdot S_0. \quad (3.13)$$

Therefore in the case of a 1<sup>st</sup>-order low-pass filtered white noise, the undersampling factor is equal to the ratio of the equivalent noise bandwidth and the Nyquist frequency [1]

$$N_{under} = \frac{\pi/2 f_c}{f_s/2} = \pi f_c T_s. \quad (3.14)$$

The output sample-and-held signal PSD is then given by [1]

$$S_{out}(f) = \left(\frac{T_h}{T_s}\right)^2 \cdot \pi f_c T_s \cdot S_0 \cdot \text{sinc}^2(\pi f T_h) \cong \pi f_c T_s \cdot S_0 \cdot \text{sinc}^2(\pi f T_s) \quad (3.15)$$

since for ideal sampling we assume that  $T_t \ll T_h$ ,  $T_h \cong T_s$ , and hence  $T_h/T_s \cong 1$ .

The sample-and-held output PSD is plotted in Figure 3.7.

Note that the equivalent noise bandwidth  $B_n$  of  $S_{out}$ , defined as the bandwidth of an ideally low-pass filtered white noise having the same value at the origin ( $f = 0$ ) and the same power, is equal to the Nyquist frequency  $f_s/2$

$$B_{neq} = \frac{1}{2S_{out}(0)} \cdot \int_{-\infty}^{+\infty} S_{out}(f) \cdot df = \frac{2B_n S_0}{4B_n S_0 T_s} = \frac{f_s}{2}. \quad (3.16)$$

This means that the aliasing introduced by the ideal sample-and-hold operation on a broadband white noise ( $B_n > f_s/2$ ) transposes all of its power into the baseband ( $|f| \leq f_s/2$ ) resulting in an increase of the PSD equal to undersampling factor  $2B_n/f_s$ . This is illustrated in Fig. Figure 3.8.

We will now validate the above results using the Spectre simulator.

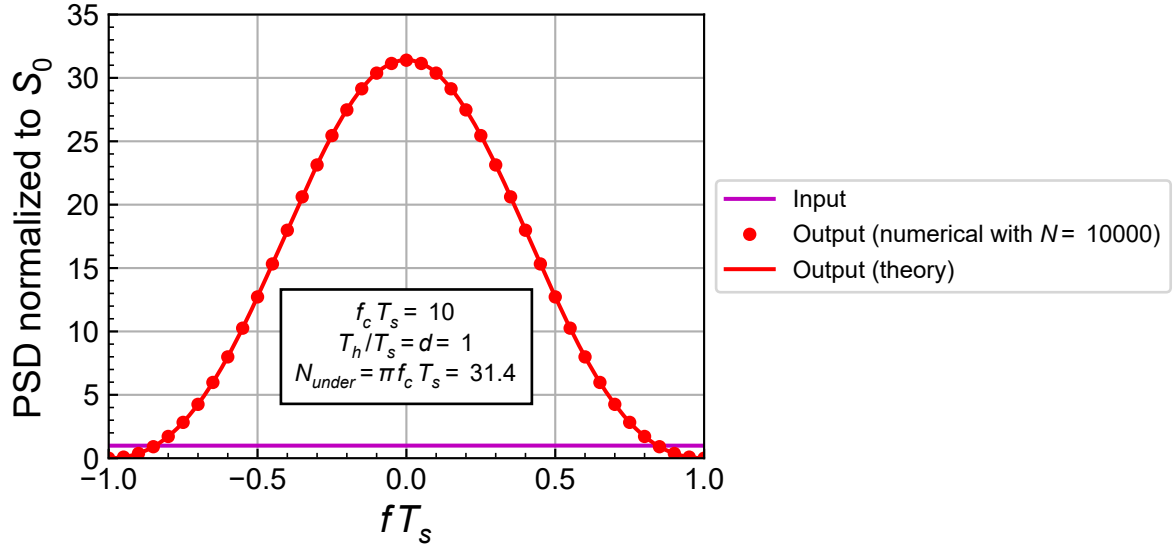


Figure 3.7: Sampled white noise PSD.

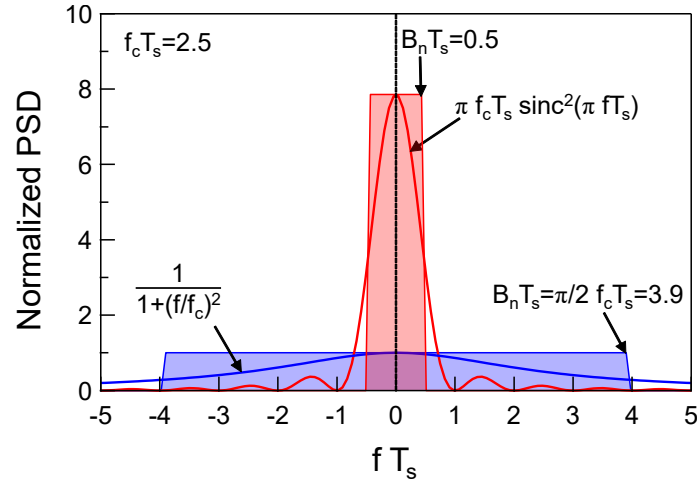


Figure 3.8: Equivalent noise bandwidth of a sampled white noise.

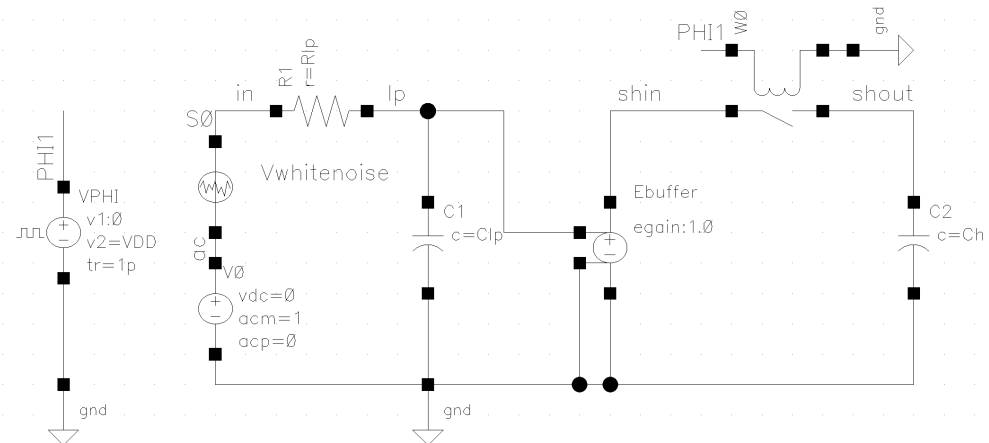


Figure 3.9: Circuit used for the simulation of noise sampling in Cadence.

### 3.2.2 Simulations

The ideal sampling has been simulated with Spectre RF using the circuit shown in Figure 3.9.

The circuit shown in Figure 3.9 basically implements the block diagram of Fig. Figure 3.1. The noise source generates an ideal white noise having a power spectral density (PSD)  $S_0$ , which is then low-pass filtered by the  $R_1 C_1$  circuit having a cut-off frequency  $f_c = 1/(2\pi R_{lp} C_{lp})$ . The output of the low-pass filter is then buffered by an ideal voltage follower generating the input to the sample-and-hold. The latter is made of an ideal switch controlled by the phas  $\Phi_1$ . The noise voltage is the sampled on the hold capacitor  $C$ .

In the example below the PSD of the white noise source has been set to that of the series resistor  $R_{lp}$

$$S_0 = 4kTR_{lp} \quad (3.17)$$

where  $k$  is the Boltzmann constant and  $T$  the absolute tempearture. We could have used the thermal noise produced by the resistor  $R_1$ , however we then cannot change the noise level without changing the noise bandwidth. In addition, using a separate noise source allows to also simulate flicker noise as shown below. Of course the resistor  $R_1$  has been set as noiseless.

The design variables for the simulation have been set according to Figure 3.10.

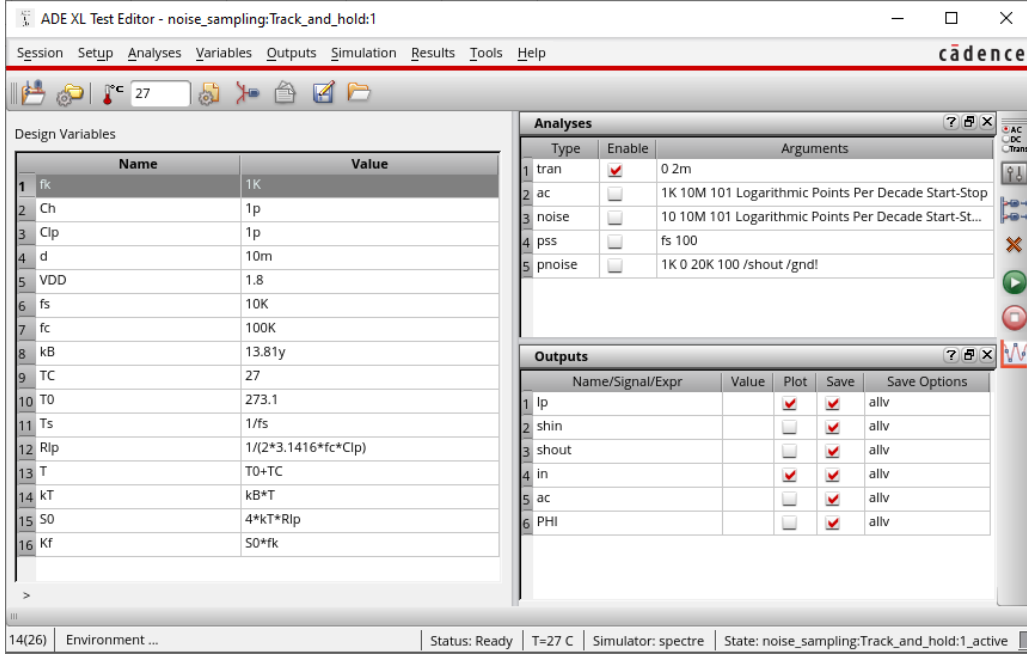


Figure 3.10: Design variables used for the simulations.

The parameter for the simulation and the model calculation are given in Table 3.1. Note that the duty cycle is defined as

$$d \triangleq \frac{T_h}{T_s} = 1 - D, \quad (3.18)$$

$$D \triangleq \frac{T_t}{T_s} = 1 - d, \quad (3.19)$$

Note that this is not the same definition as in the Spectre simulation where  $d$  and  $D$  have been swapped. We keep the above definition to be consistent with the Chapter on noise and offset reduction techniques using autozero.

Table 3.1: Parameters used for the simulation of the white noise with Spectre.

| Parameter                  | Value     | Unit          |
|----------------------------|-----------|---------------|
| $f_s$                      | 10        | $kHz$         |
| $f_c$                      | 100       | $kHz$         |
| $f_k$                      | 1         | $kHz$         |
| $D \triangleq T_t/T_s$     | 0.01      | -             |
| $d \triangleq T_h/T_s$     | 0.99      | -             |
| $T_s$                      | 100       | $\mu s$       |
| $T_t$                      | 1         | $\mu s$       |
| $T_h$                      | 99        | $\mu s$       |
| $f_c T_s$                  | 10        | -             |
| $f_k T_s$                  | 0.1       | -             |
| $R_{lp}$                   | 2         | $M\Omega$     |
| $S_0 = 4kTR_{lp}$          | 2.638e-14 | $V^2/Hz$      |
| $B_n$                      | 157       | $kHz$         |
| $V_n$                      | 64        | $\mu V_{rms}$ |
| $N_{under} = \pi f_c T_s$  | 31.4      | -             |
| $N_{duty} = d^2 N_{ideal}$ | 30.8      | -             |

We see that the undersampling factor  $N_{under} = \pi f_c T_s$  is equal to 31.4 and if we account for the 1 % duty cycle it decreases to

$$\left(\frac{T_h}{T_s}\right)^2 \cdot \pi f_c T_s = d^2 \cdot \pi f_c T_s, \quad (3.20)$$

which is equal to 30.8.

These numbers will now be compared to the results obtained from the Spectre simulations. In the simulation, we have taken 1000 subbands to get as close to the theoretical result as possible.

We first check the low-pass filter transfer function and the input noise PSD by running an .AC and .NOISE simulations. The transfer function and white noise PSD are shown in Figure 3.11.

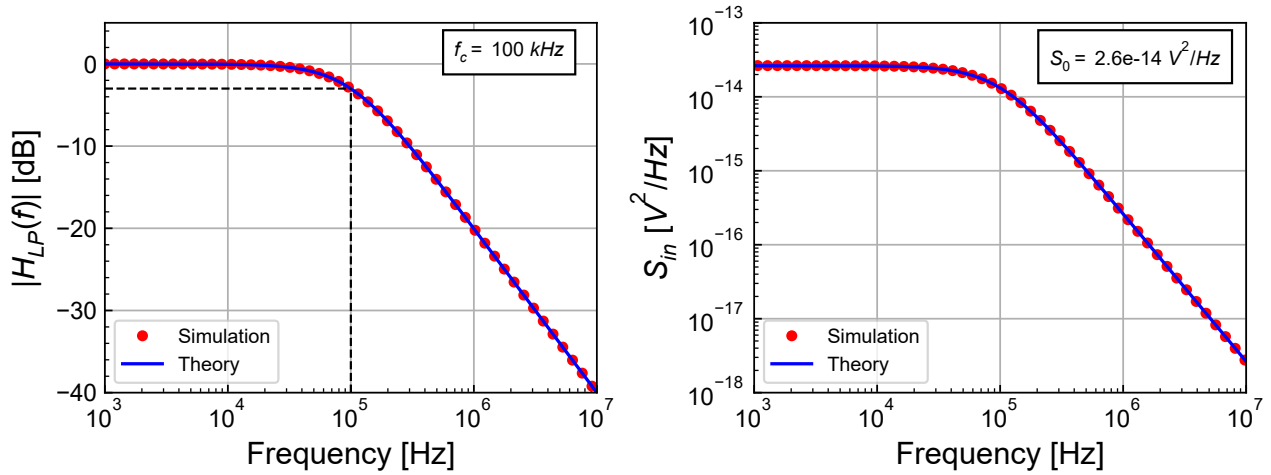


Figure 3.11: Spectre simulation of the transfer function and LP filtered white noise PSD.

We see a perfect agreement between simulations and model which is expected at this point. Before simulating the output PSD, we can simulate the noise in the time domain using a transient noise simulation. The results are shown in Figure 3.12.

Figure 3.12 nicely illustrates how the input white noise (in blue in Figure 3.12) is sampled at the beginning of each period and then held for the rest of the period. The spikes are simply due to the

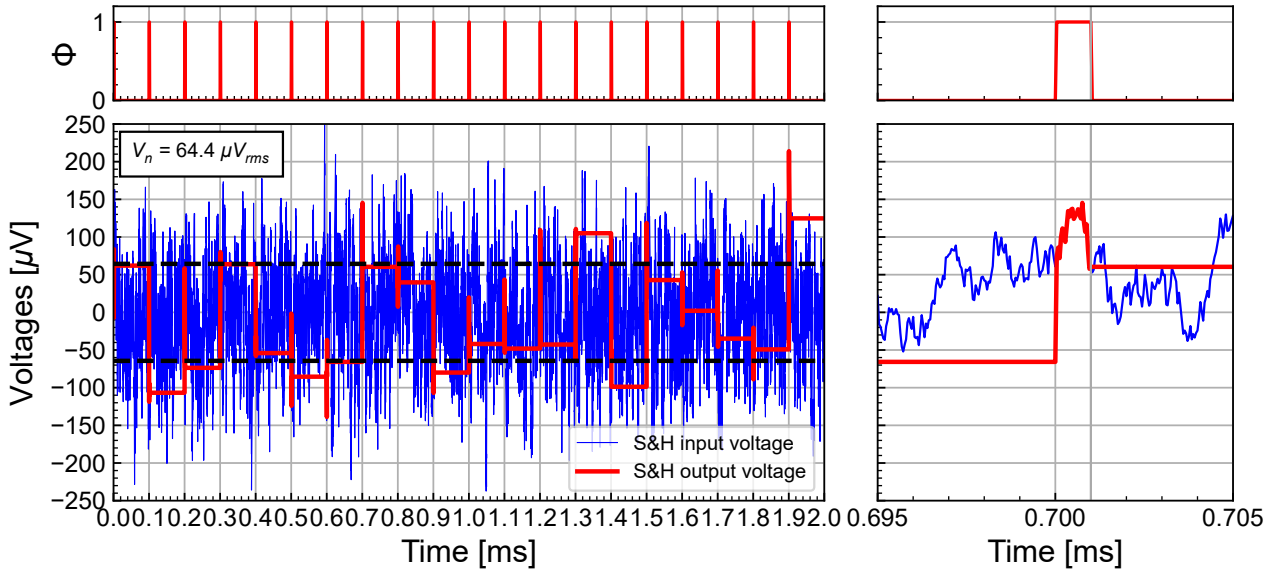


Figure 3.12: Spectre transient noise simulations showing the input and output waveforms.

output signal following the input signal during the short track time as illustrated in the zoomed figure on the right of Figure 3.12. The black dashed lines correspond to the noise rms voltage given by

$$V_n^2 = B_n \cdot S_0, \quad (3.21)$$

corresponding to an rms voltage  $V_n = 64 \mu V_{rms}$ .

We now will simulate the PSD of the output signal using a `pss` and `pnoise` analysis with the parameters as shown in Figure 3.13. (check the link to Ken Kundert's web site [The Designer's Guide](#) where you will find many useful information about this topic [8]).

The Spectre simulation result is shown in Figure 3.14. Note that the plot of Figure 3.14 shows the input and output PSD normalized to  $S_0$ . We see a perfect match between the model and the simulations, confirming the increase of the PSD by more than a factor 30!

### 3.3 Flicker noise sampling

In this Section we will show that sampling is also aliasing the flicker noise PSD but the spectrum folded back to the baseband is much smaller than that of the broadband white noise.

#### 3.3.1 Theory

We will express the flicker noise PSD as

$$S_{flicker}(f) = \frac{K_f}{|f|^\alpha}, \quad (3.22)$$

where we will consider that  $\alpha = 1$ . We will set the flicker noise level  $K_f$  so that the value of the flicker noise is equal to that of the white noise PSD  $S_0$  we used above at a frequency called the corner frequency  $f_k$

$$K_f = S_0 \cdot f_k. \quad (3.23)$$

In the S&H circuit, the flicker noise is then also low-pass filtered so that the PSD of the S&H input voltage is

$$S_{in}(f) = \frac{S_{flicker}(f)}{1 + (f/f_c)^2} = \frac{K_f}{|f|} \cdot \frac{1}{1 + (f/f_c)^2}. \quad (3.24)$$

**Choosing Analyses -- ADE L (1)@iclabsrv3**

Analysis: ☐ tran ☐ dc ☐ ac ☐ noise  
☐ xf ☐ sens ☐ dcmatch ☐ acmatch  
☐ stb ☐ pz ☐ sp ☐ envlp  
☒ pss ☐ pac ☐ pstb ☐ pnoise  
☐ pxf ☐ psp ☐ qpss ☐ qpac  
☐ qpnoise ☐ qpxf ☐ qpss ☐ hb  
☐ hbac ☐ hbstb ☐ hbnoise ☐ hbsp  
☐ hbxf

Engine: ☒ Shooting ☐ Harmonic Balance

Periodic Steady State Analysis

Fundamental Tones

| # | Name | Expr     | Value | Signal | SrcId |
|---|------|----------|-------|--------|-------|
| 2 |      | 1/(Ts-0) | 10K   | Large  | VPHI  |

1/(Ts-0) 10K Large VPHI

Clear/Add Delete Update From Hierarchy

Beat Frequency ☒ fs Auto Calculate ☐  
Beat Period ☐

Output harmonics  
Number of harmonics 100

Accuracy Defaults (errpreset)  
☒ conservative ☐ moderate ☐ liberal

Transient-Aided Options  
Run transient? ☒ Yes ☐ No ☐ Decide automatically  
Detect Steady State ☐ Stop Time (tstab) 1m  
Save Initial Transient Results (saveinit) ☐ no ☐ yes

Dynamic Parameter ☐

Oscillator ☐

Sweep ☐  
New Initial Value For Each Point (restart) ☐ no ☐ yes

Loadpull ☐

Enabled ☒ Options...

**Choosing Analyses -- ADE L (1)@iclabsrv3**

Analysis: ☐ tran ☐ dc ☐ ac ☐ noise  
☐ xf ☐ sens ☐ dcmatch ☐ acmatch  
☐ stb ☐ pz ☐ sp ☐ envlp  
☐ pss ☐ pac ☐ pstb ☒ pnoise  
☐ pxf ☐ psp ☐ qpss ☐ qpac  
☐ qpnoise ☐ qpxf ☐ qpss ☐ hb  
☐ hbac ☐ hbstb ☐ hbnoise ☐ hbsp  
☐ hbxf

Periodic Noise Analysis

PSS Beat Frequency (Hz)

Multiple pnoise ☒

| Name    | Input | Output     | Relharm | Enable                              |
|---------|-------|------------|---------|-------------------------------------|
| pnoise1 |       | shout-gnd! |         | <input checked="" type="checkbox"/> |
| pnoise2 |       | shin-gnd!  |         | <input checked="" type="checkbox"/> |
| pnoise3 |       |            |         | <input type="checkbox"/>            |
| pnoise4 |       |            |         | <input type="checkbox"/>            |
| pnoise5 |       |            |         | <input type="checkbox"/>            |
| pnoise6 |       |            |         | <input type="checkbox"/>            |

SweepType2 default Sweep is currently absolute

Output Frequency Sweep Range (Hz)  
Start-Stop Start 0 Stop 20k

Sweep Type  
Linear Step Size 100  
Number of Steps

Add Specific Points ☐  
Update From pnoise 1

Noise Figure ☐

Output2  
voltage Positive Output Node /shin Select  
Negative Output Node /gnd! Select

Noise Type2 timeaverage  
Timeaverage: single-sided spectrum and harmonic-referred (modulated) noise analysis  
Contribution Type:  
☒ USB ☐ AM ☐ PM ☐ AM&PM ☐ ALL(AM,PM,USB,LSB)  
Noise Separation ☐  
Separate noise into source and gain

Enabled ☒ Options...

Figure 3.13: pnoise set-up with 1000 subbands (not shown).

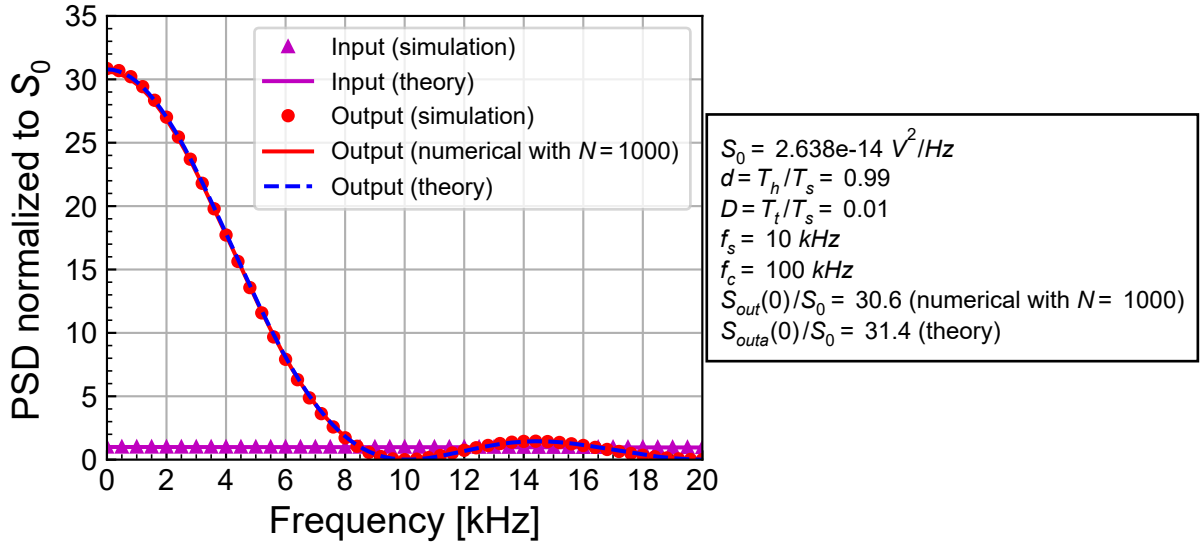


Figure 3.14: Spectre simulation of the output PSD for a LP filtered white noise input PSD.

The ideally sampled flicker noise is then obtained as

$$S_{V_s}(f) = \sum_{n=-\infty}^{+\infty} S_{in}\left(f - \frac{n}{T_s}\right) = K_f T_s \cdot \sum_{n=-\infty}^{+\infty} \frac{1}{|f T_s - n|} \cdot \frac{1}{1 + \left(\frac{f T_s - n}{f_c T_s}\right)^2}. \quad (3.25)$$

The shifted version of the input PSD

$$S_{in}(f T_s - n) = \frac{K_f T_s}{|f T_s - n|} \cdot \frac{1}{1 + \left(\frac{f T_s - n}{f_c T_s}\right)^2} \quad (3.26)$$

normalized to  $K_f T_s = S_0 \cdot f_c T_s$  are plotted in Figure 3.15 for  $f_c T_s = 10$  and  $n = -2, -1, 0, +1, +2$ .

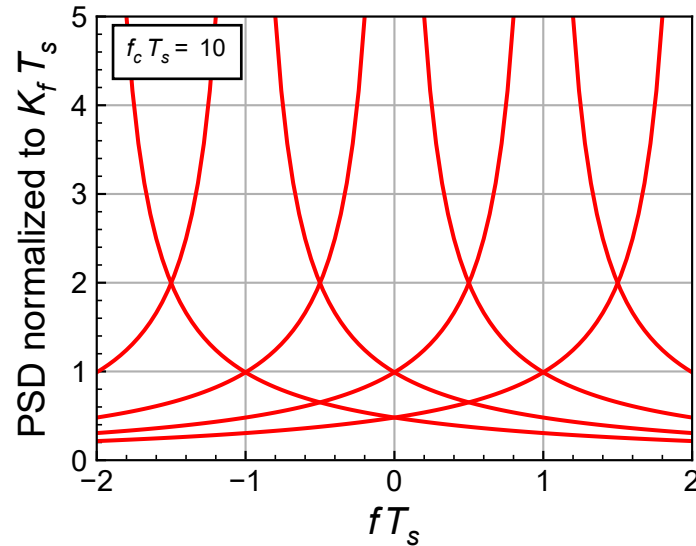


Figure 3.15: Shifted versions of the input flicker noise PSD.

From the above plot we see that we can decompose the ideally sampled flicker noise PSD into the input PSD plus a foldover component [1]

$$S_{V_s}(f) = S_{flicker}(f) + S_{fold}(f), \quad (3.27)$$

where  $S_{fold}(f)$  is the foldover term corresponding to all the spectra that are aliased into the baseband [1]

$$S_{fold}(f) \triangleq K_f T_s \cdot \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \frac{1}{|f T_s - n|} \cdot \frac{1}{1 + \left(\frac{f T_s - n}{f_c T_s}\right)^2}. \quad (3.28)$$

The foldover component normalized to  $K_f \cdot T_s$  computed for  $N = 100$  terms is plotted in Figure 3.16.

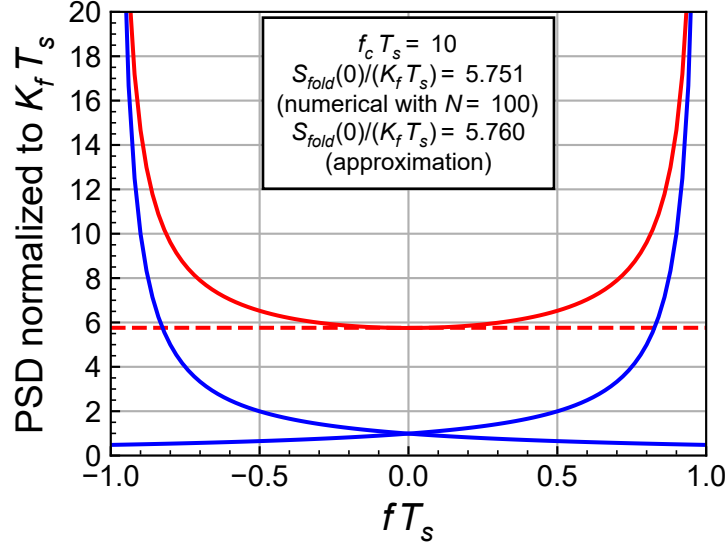


Figure 3.16: Flicker noise foldover component.

From Figure 3.16, we see that the foldover component at low frequency can be considered as constant and equal to  $S_{fold}(f = 0)$ . It can be shown that this value can be calculated analytically (using Mathematica for example) with the help of the digamma function  $\Psi(z)$

$$\begin{aligned} S_{fold}(f = 0) &= K_f T_s \cdot [2\gamma + \Psi(1 + j f_c T_s) + \Psi(1 - j f_c T_s)] \\ &= 2K_f T_s \cdot [\gamma + \Re\{\Psi(1 + j f_c T_s)\}], \end{aligned} \quad (3.29)$$

where  $\gamma \cong 0.577216$  is the Euler constant and  $\Psi(z)$  is the [Psi or Digamma function](#).

For  $f_c T_s > 2$  the foldover component given by (3.29) can be approximated by [1]

$$S_{fold} \cong 2K_f T_s \cdot [\gamma + \ln(f_c T_s)] \quad (3.30)$$

with a relative error smaller than 4%. We see from (3.30) that the foldover component now increases with the log of the cut-off frequency instead of linearly as is the case for the white noise.

The foldover component given by (3.29) normalized to  $K_f T_s$  is plotted versus  $f_c T_s$  in Figure 3.17. It is compared to the result of the numerical summation with  $N_{term}$  terms and to the approximation.

We see that the approximation is excellent for  $f_c T_s > 2$ . The dashed black line corresponds to the foldover of the white noise which increases linearly with  $f_c T_s$ .

The above PSD correspond to the ideally sampled flicker noise. After the hold function it must be multiplied by  $\text{sinc}^2(\pi f T_h)$ . The PSD of the sample-and-held flicker noise can then be approximated by [1]

$$S_{out}(f) \cong \left(\frac{T_h}{T_s}\right)^2 \cdot \text{sinc}^2(\pi f T_h) \cdot K_f T_s \cdot \left\{ \frac{1}{f T_s} + 2[\gamma + \ln(f_c T_s)] \right\} \quad (3.31)$$

The above approximation (3.31) is compared in Figure 3.18 to the numerical result computed for  $N$  terms.

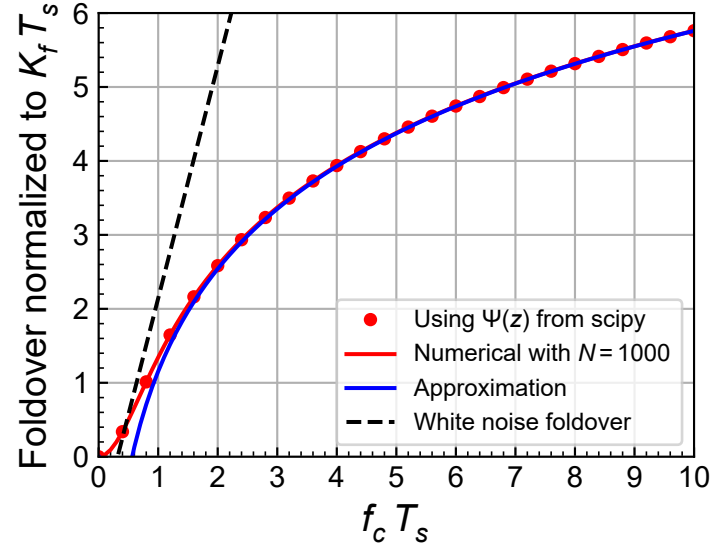


Figure 3.17: Exact expression of the flicker noise foldover component (3.29) versus  $f_c T_s$  compared to the approximation (3.30).

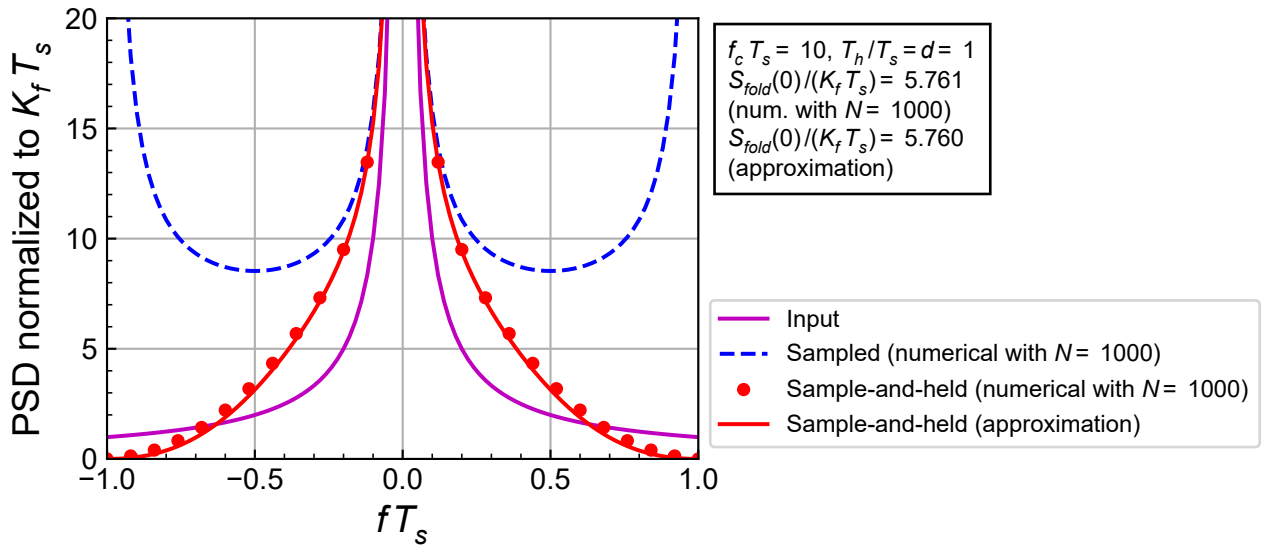


Figure 3.18: PSD of the sample-and-held flicker noise.

We see that the simple model (3.31) gives a very good approximation of the sample-and-held flicker noise in the baseband.

The above approximation will now be compared with Spectre simulations.

### 3.3.2 Simulations

The simulations have been performed with the same circuit of Figure 3.9 used for the white noise except that the white noise source has been replaced by a flicker noise source. The level of the flicker noise has been set such that it is equal to the white PSD used above for a frequency of  $f_k = 1 \text{ kHz}$  which will correspond to the corner frequency. The simulations have been performed with the parameters shown in Table 3.2.

Table 3.2: Parameters used for the Spectre simulation for a flicker noise input.

| Parameter              | Value     | Unit                   |
|------------------------|-----------|------------------------|
| $f_s$                  | 10        | $\text{kHz}$           |
| $f_c$                  | 100       | $\text{kHz}$           |
| $f_k$                  | 1         | $\text{kHz}$           |
| $D \triangleq T_t/T_s$ | 0.01      | -                      |
| $d \triangleq T_h/T_s$ | 0.99      | -                      |
| $T_s$                  | 100       | $\mu\text{s}$          |
| $T_t$                  | 1         | $\mu\text{s}$          |
| $T_h$                  | 99        | $\mu\text{s}$          |
| $f_c T_s$              | 10        | -                      |
| $f_k T_s$              | 0         | -                      |
| $R_{lp}$               | 2         | $M\Omega$              |
| $K_f$                  | 2.638e-11 | $\text{V}^2$           |
| $K_f T_s$              | 2.638e-15 | $\text{V}^2/\text{Hz}$ |
| $S_{fold}(0)$          | 1.519e-14 | $\text{V}^2/\text{Hz}$ |
| $(T_h/T_s)^2$          | 0.0001    | -                      |

The low-pass transfer function and the PSD of the input flicker noise are shown in Figure 3.19.

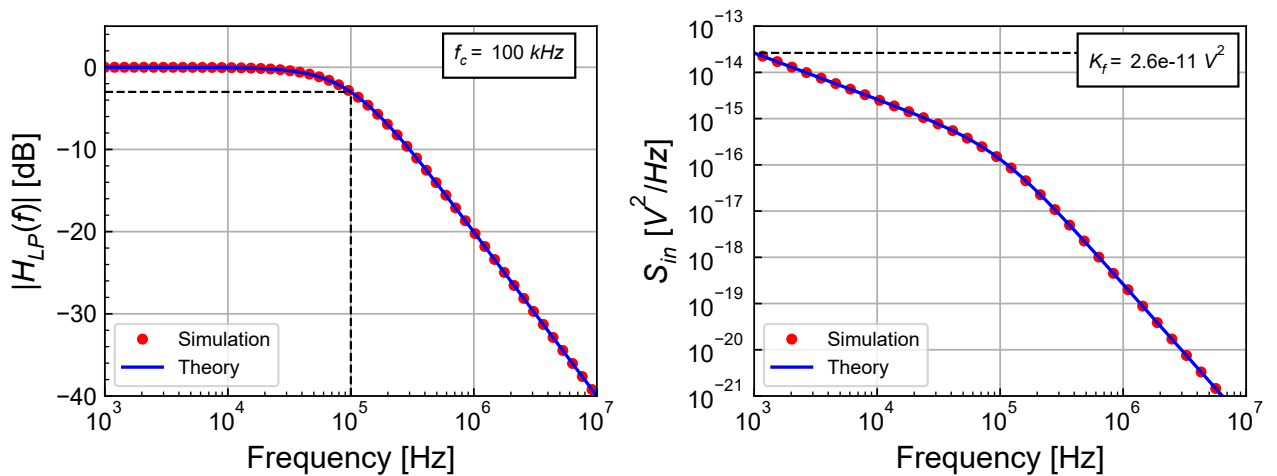


Figure 3.19: Spectre simulation of the transfer function and LP filtered flicker noise PSD.

The low-pass filtered flicker noise input PSD corresponds to what is expected from the theory.

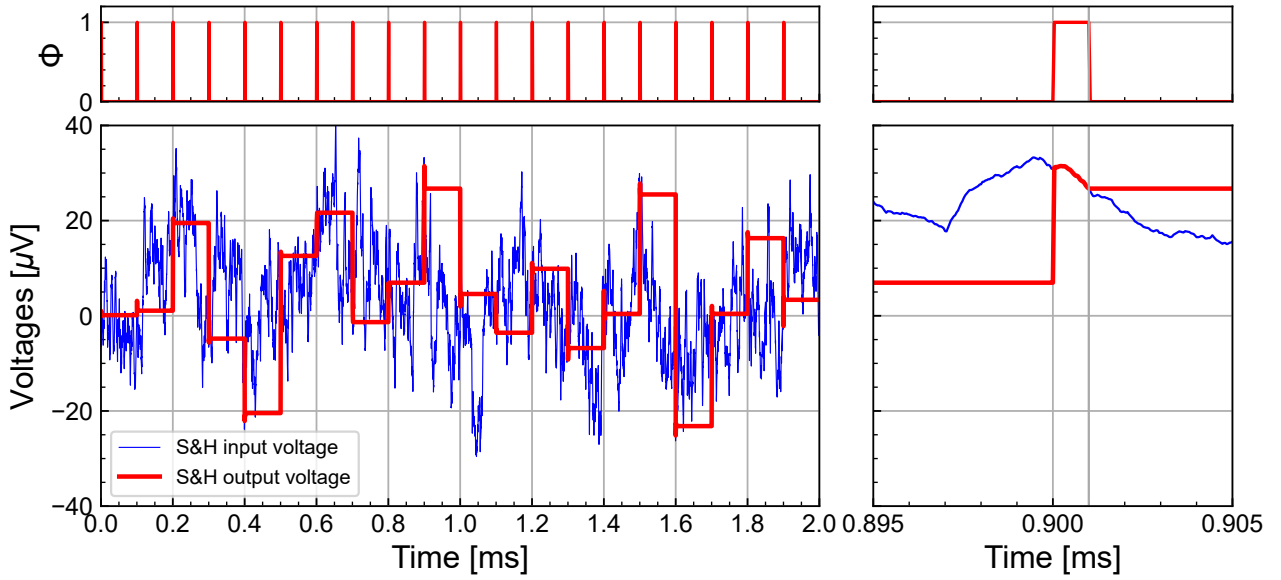


Figure 3.20: Spectre transient noise simulations showing the input and output waveforms for a flicker noise input.

We can also perform a transient noise analysis for the flicker noise resulting in the waveforms shown in Figure 3.20.

We see that the flicker noise in the time domain is much less busy and of less amplitude than the white noise.

The next step is to compare the simulated sample-and-held flicker noise to the theory which is done in Figure 3.21.

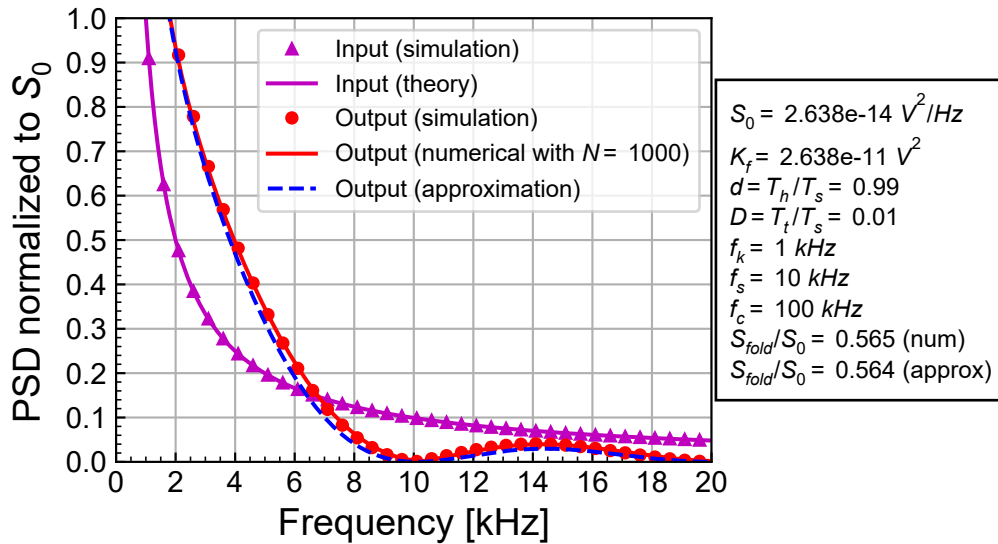


Figure 3.21: Spectre simulation of the output PSD for a LP filtered flicker noise input PSD.

We see a very good agreement between the sample-and-held flicker noise model and the simulation results.

We are actually most interested in the effect of sampling in the baseband (or Nyquist band) which is shown in Figure 3.22.

We see a very good match between the model and the simulations.

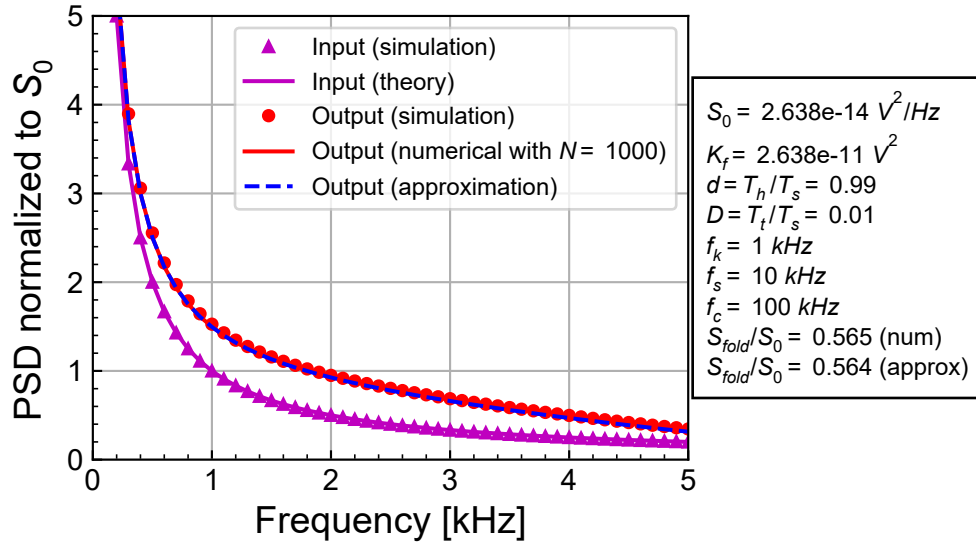


Figure 3.22: Spectre simulation of the output PSD for a LP filtered flicker noise input PSD.

We can also check the foldover component by substracting the input PSD. This is done below.

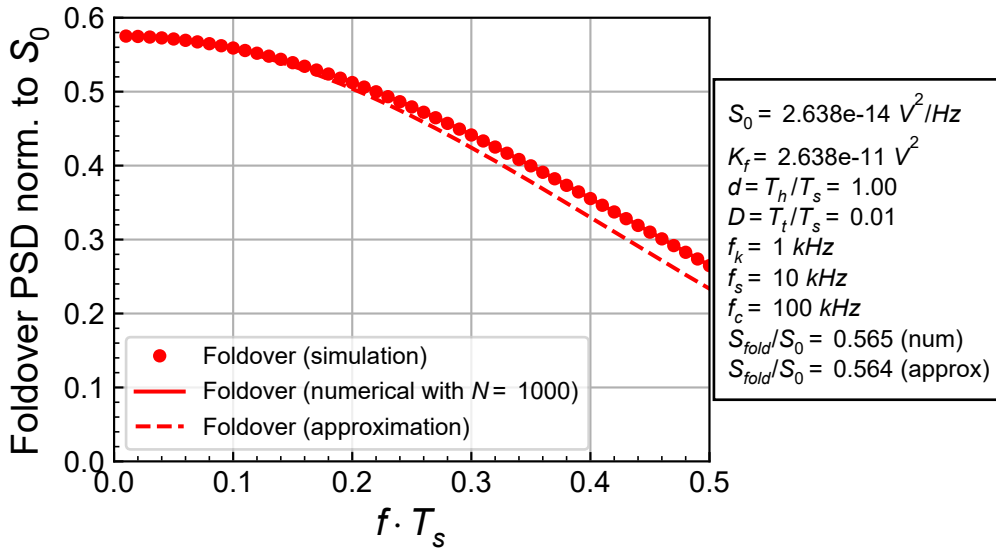


Figure 3.23: Spectre simulation of the foldover component for a LP filtered flicker noise input PSD.

In the next section, we will combine white and flicker and evaluate the output PSD.

## 3.4 Combined white and flicker noise sampling

### 3.4.1 Theory

We can now combine the above result to model the ideal sample-and-hold process of a noise including both white and flicker noise. Since the physical processes generating the white noise (thermal noise or shot noise) and the flicker noise are very different, they can be considered as *uncorrelated*. The PSD of the noise source can then be written as

$$S_n(f) = S_0 \cdot \left(1 + \frac{f_k}{|f|}\right), \quad (3.32)$$

where  $S_0$  is the white noise PSD and  $f_k$  is the *corner frequency* defined as the frequency at which the flicker noise becomes equal to the white noise PSD. As before we consider that the input noise of the S&H is low-pass filtered

$$S_{in}(f) = \frac{S_n(f)}{1 + (f/f_c)^2} = \frac{S_0}{1 + (f/f_c)^2} \cdot \left(1 + \frac{f_k}{|f|}\right). \quad (3.33)$$

As we have done for the flicker noise, it is better to express the S&H output PSD in the baseband in terms of the fundamental components and a foldover term

$$S_{out}(f) = S_{bb}(f) + S_{fold}(f), \quad (3.34)$$

where

$$S_{bb}(f) = \left(\frac{T_h}{T_s}\right)^2 \text{sinc}^2(\pi f T_h) \cdot S_{in}(f) = d^2 \cdot \text{sinc}^2(\pi d f T_s) \cdot S_{in}(f) \quad (3.35)$$

is the baseband component and

$$\begin{aligned} S_{fold}(f) &\cong \left(\frac{T_h}{T_s}\right)^2 \text{sinc}^2(\pi f T_h) \cdot S_0 \cdot \{\pi f_c T_s - 1 + 2f_k T_s \cdot [\gamma + \ln(f_c T_s)]\} \\ &= d^2 \cdot \text{sinc}^2(\pi d f T_s) \cdot S_0 \cdot \{\pi f_c T_s - 1 + 2f_k T_s \cdot [\gamma + \ln(f_c T_s)]\} \end{aligned} \quad (3.36)$$

is the foldover component due to the aliasing of both the white and flicker noise harmonics into the baseband. Note that the  $-1$  term in the foldover component comes from the fact that the  $n = 0$  term is already included in the baseband term and has to be subtracted from the foldover not to account for it twice.

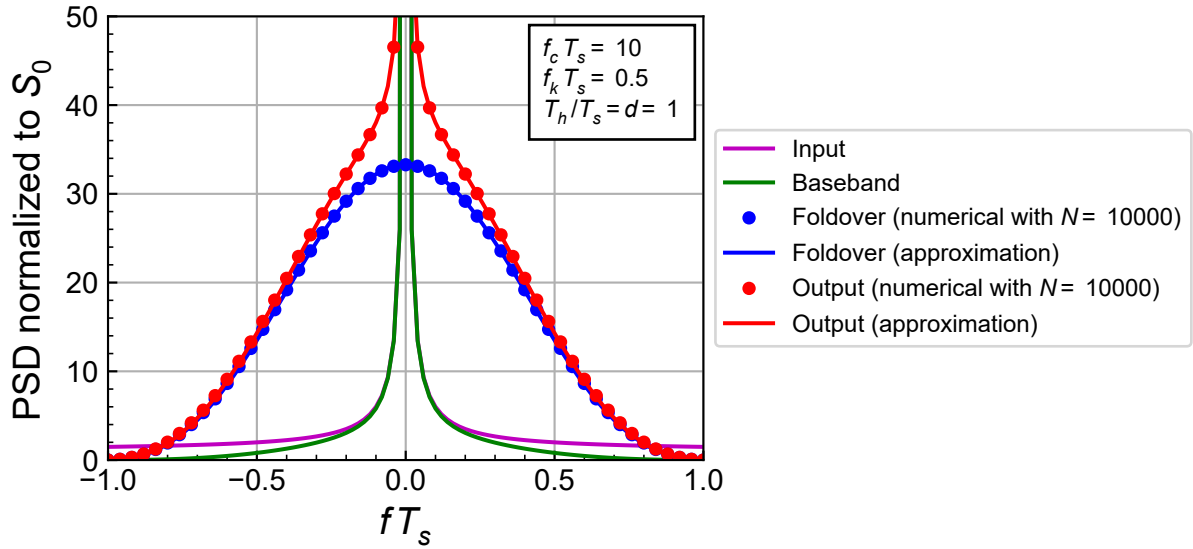


Figure 3.24: Output PSD for a combined white and flicker noise input PSD.

The above expression of the output PSD will now be validated by Spectre simulations.

### 3.4.2 Simulations

Table 3.3: Parameters used for the Spectre simulation of the combined white and flicker noise.

| Parameter | Value | Unit  |
|-----------|-------|-------|
| $f_s$     | 10    | $kHz$ |

Table 3.3: Parameters used for the Spectre simulation of the combined white and flicker noise.

| Parameter              | Value     | Unit          |
|------------------------|-----------|---------------|
| $f_c$                  | 100       | $kHz$         |
| $f_k$                  | 1         | $kHz$         |
| $D \triangleq T_t/T_s$ | 0.01      | -             |
| $d \triangleq T_h/T_s$ | 0.99      | -             |
| $T_s$                  | 100       | $\mu s$       |
| $T_t$                  | 1         | $\mu s$       |
| $T_h$                  | 99        | $\mu s$       |
| $f_c T_s$              | 10        | -             |
| $f_k T_s$              | 0         | -             |
| $R_{lp}$               | 2         | $M\Omega$     |
| $S_0 = 4kTR_{lp}$      | 2.638e-14 | $V^2/Hz$      |
| $B_n$                  | 157       | $kHz$         |
| $V_n$                  | 64        | $\mu V_{rms}$ |
| $K_f$                  | 2.638e-11 | $V^2$         |
| $K_f T_s$              | 2.638e-15 | $V^2/Hz$      |
| $S_{fold}(0)$          | 1.49e-14  | $V^2/Hz$      |

The transfer function and the input PSD are shown in Figure 3.26. The input noise PSD has the expected corner and cut-off frequency and matches the theory.

Figure 3.27 shows the input and output waveforms resulting from a transient noise simulation.

The simulated output PSD is compared to the theory and approximation in Figure 3.28.

The approximation of the sample-and-held output signal PSD perfectly matches the simulations. We can have a closer look at the baseband which is shown in Figure 3.29.

Again the approximation gives almost a perfect match to the simulations. Finally we can also plot the PSD in a log-log scale which is shown in Figure 3.30.

In the next section, we will proceed with the more realistic case of track-and-hold signals.

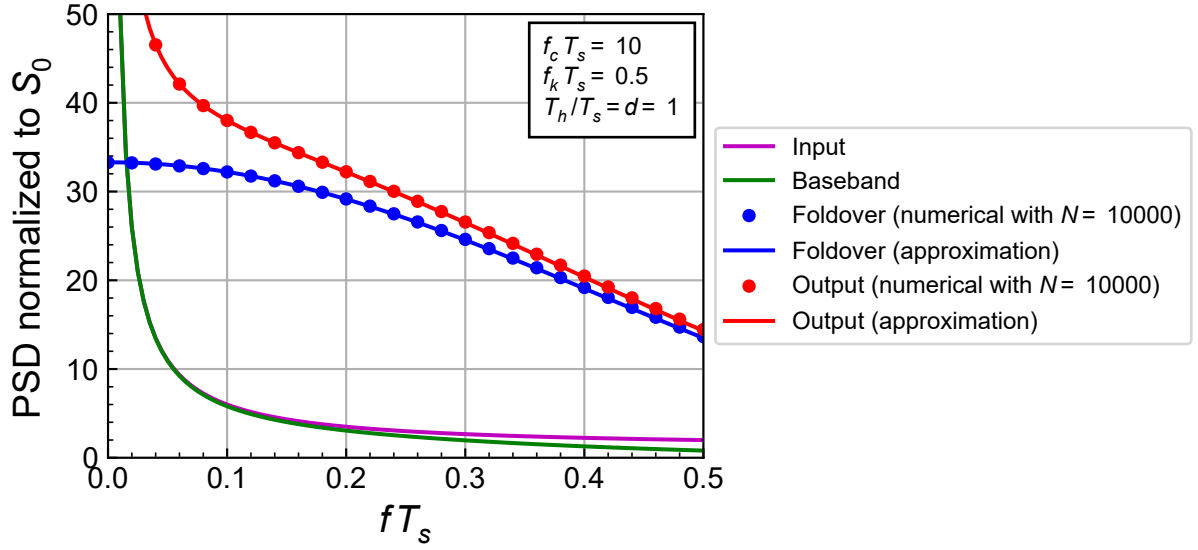


Figure 3.25: Output PSD for a combined white and flicker noise input in the baseband.

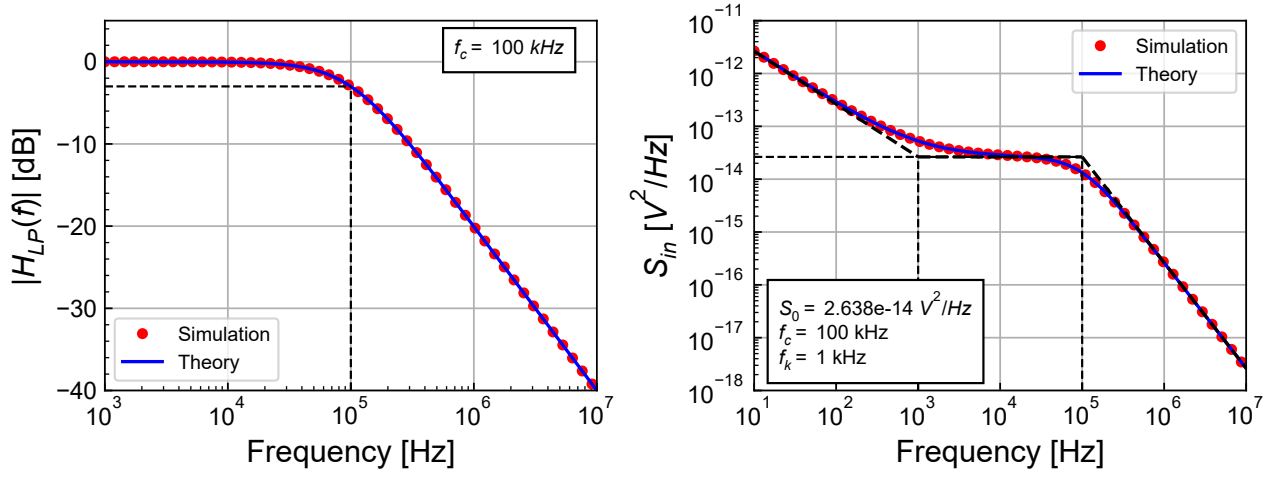


Figure 3.26: Spectre simulation of the transfer function and LP filtered combined input noise PSD.

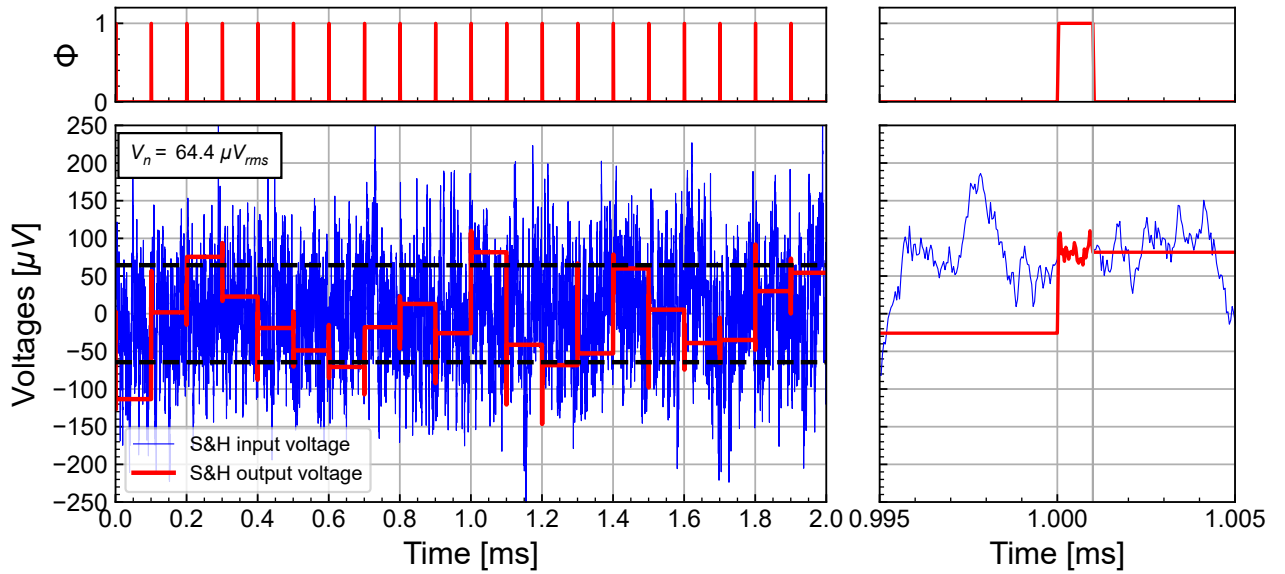


Figure 3.27: Spectre transient noise simulations showing the input and output waveforms for a combined noise input.

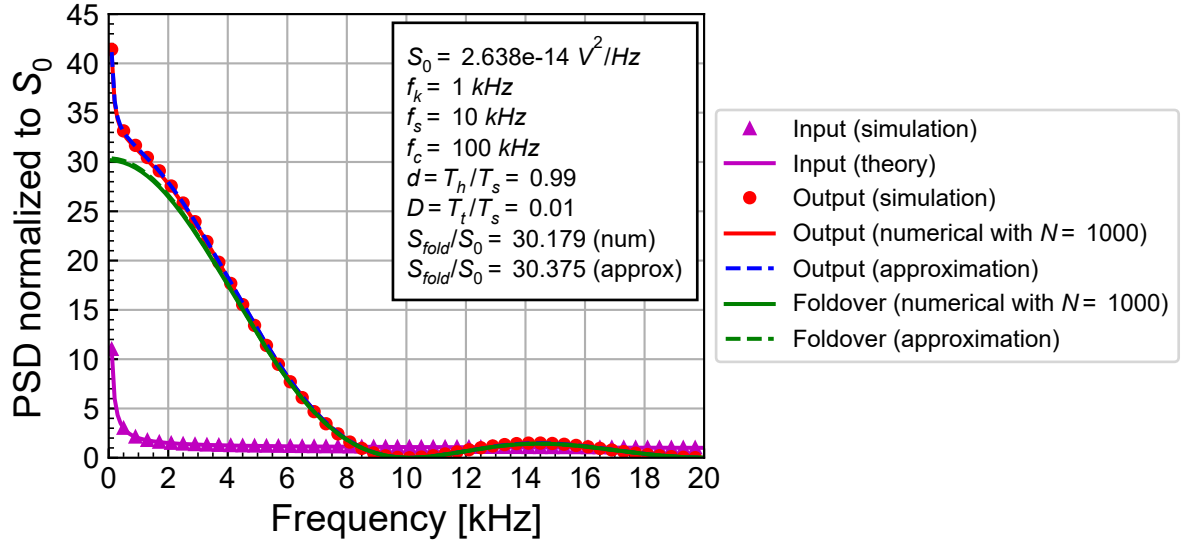


Figure 3.28: Spectre simulation of the output PSD for a LP filtered flicker noise input PSD.

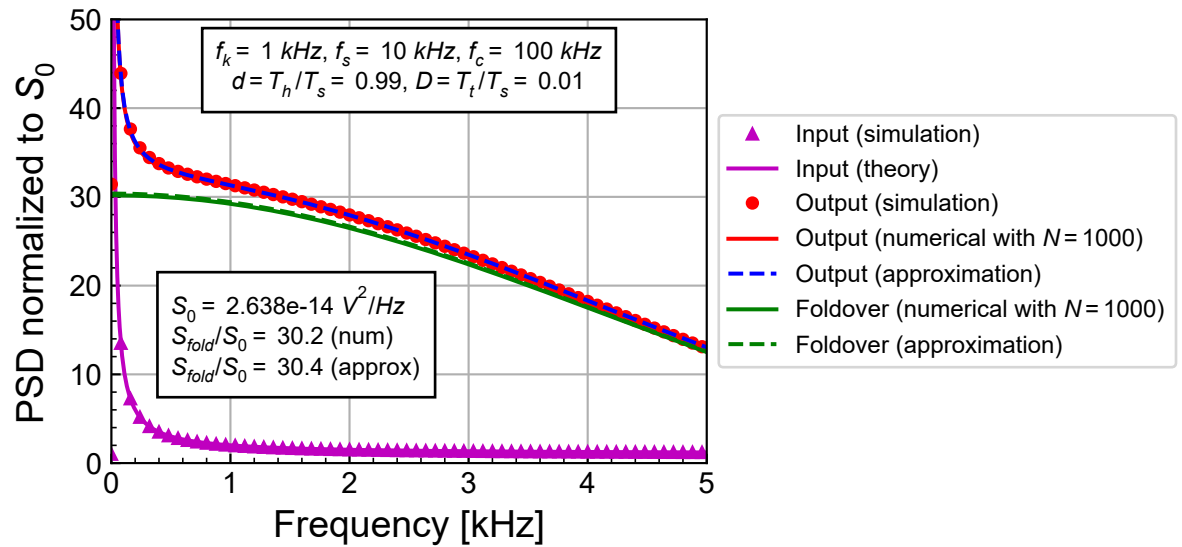


Figure 3.29: Spectre simulation of the output PSD for a LP filtered combined noise input PSD in the baseband.

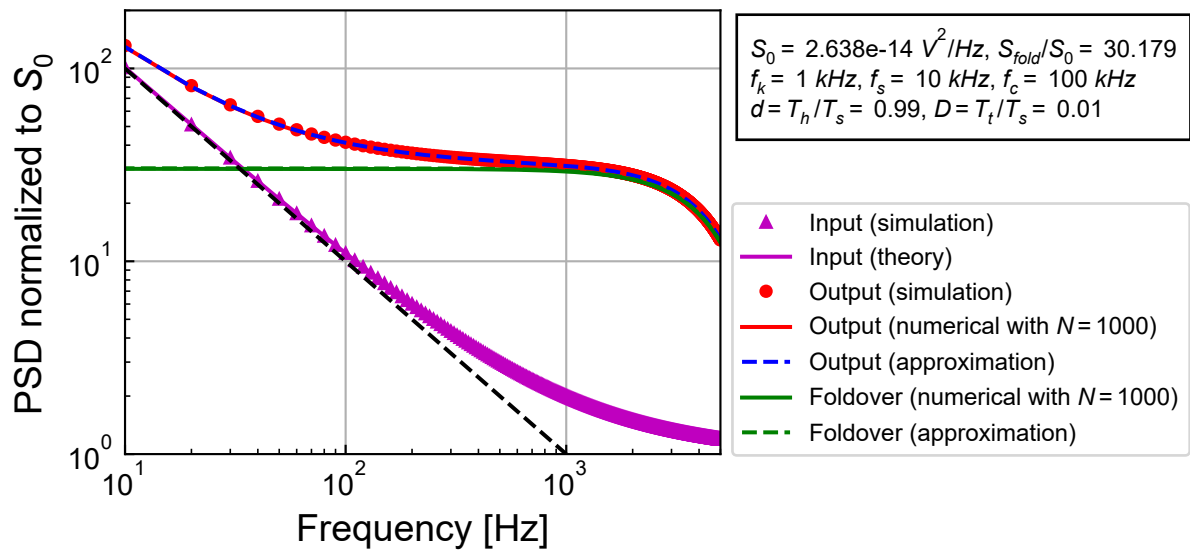


Figure 3.30: Spectre simulation of the output PSD for a LP filtered combined noise input PSD in the baseband.

## 4 Track-and-hold

### 4.1 The track-and-hold model

SC circuits often use two non-overlapping clocks which correspond to a duty cycle that is close to  $1/2$ . The ideal sample-and-hold model derived above does not account for the track period and will therefore give erroneous results. In this section we will derive the output PSD for the track-and-hold circuit without assuming that  $T_t \ll T_h$ .

Figure 4.1 shows the particular case of  $D = d = 0.5$  i.e.  $T_t = T_h = T_s/2$ .

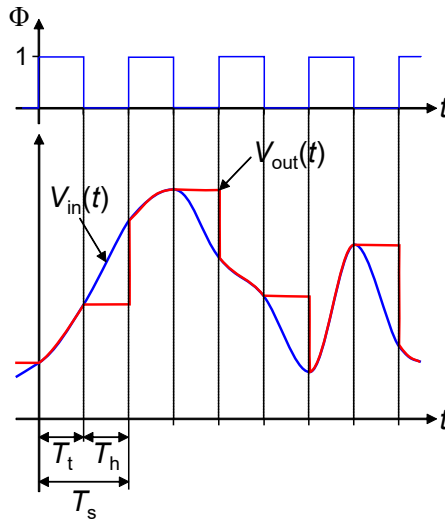


Figure 4.1: The track-and-hold waveforms assuming a duty cycle  $D = d = 0.5$  ( $T_t = T_h = T_s/2$ ).

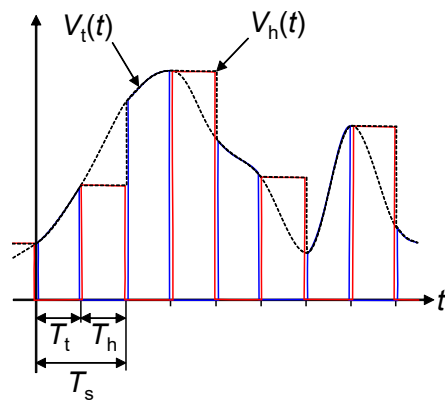


Figure 4.2: Decomposition of the signal into a track signal  $V_t(t)$  and a hold signal  $V_h(t)$

As shown in Fig. Figure 4.2, in order to analyze the track-and-hold signal, the later is decomposed into the sum of two waveforms: one for the track phase  $V_t(t)$  and one for the hold phase  $V_h(t)$  such that

$$V_{out}(t) = V_t(t) + V_h(t). \quad (4.1)$$

The track signal  $V_t(t)$  can be written as

$$V_t(t) = V_{in}(t) \cdot w(t), \quad (4.2)$$

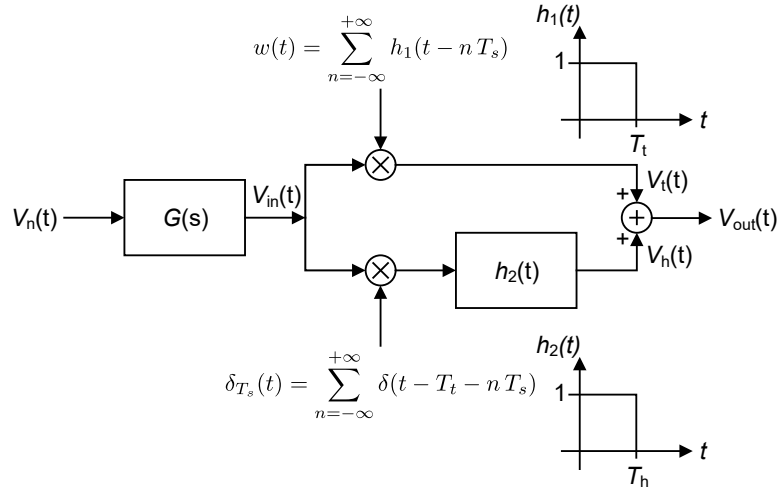


Figure 4.3: Corresponding T&amp;H block diagram.

where the window function  $w(t)$  is given by

$$w(t) = \sum_{n=-\infty}^{+\infty} h_1(t - n T_s), \quad (4.3)$$

with  $h_1(t)$  defined as

$$h_1(t) = \begin{cases} 1 & \text{for } 0 < t \leq T_t, \\ 0 & \text{otherwise.} \end{cases} \quad (4.4)$$

The hold signal  $V_h(t)$  is given by

$$\begin{aligned} V_h(t) &= h_2(t) * V_{in}(t) \cdot \sum_{n=-\infty}^{+\infty} \delta(t - T_t - n T_s) \\ &= h_2(t) * \sum_{n=-\infty}^{+\infty} V_{in}(T_t + n T_s) \delta(t - T_t - n T_s) \\ &= \sum_{n=-\infty}^{+\infty} V_{in}(T_t + n T_s) h_2(t - T_t - n T_s), \end{aligned} \quad (4.5)$$

where the  $*$  symbol corresponds to the convolution operator and  $h_2(t)$  is defined by

$$h_2(t) = \begin{cases} 1 & \text{for } 0 < t \leq T_h, \\ 0 & \text{otherwise.} \end{cases} \quad (4.6)$$

If the input signal is a stationary input noise, we cannot simply sum the PSD of the track and the hold signal. We need to account for the eventual correlation between the track and the hold period. We will now derive the output PSD in the particular case of a low-pass filtered white noise input PSD.

## 4.2 White noise sampling

### 4.2.1 Theory

If the input signal is a broadband white noise, then there is no correlation between the track and the hold period and the PSD of the output track-and-hold signal can then be written as

$$S_{out}(f) = S_t(f) + S_h(f), \quad (4.7)$$

where the track part of the output PSD is given by

$$\begin{aligned} S_t(f) &= \left(\frac{T_t}{T_s}\right)^2 \sum_{n=-\infty}^{+\infty} \text{sinc}^2\left(\frac{\pi n T_t}{T_s}\right) \cdot S_{in}\left(f - \frac{n}{T_s}\right) \\ &= D^2 \sum_{n=-\infty}^{+\infty} \text{sinc}^2(\pi n D) \cdot S_{in}\left(f - \frac{n}{T_s}\right), \end{aligned} \quad (4.8)$$

where  $D \triangleq T_t/T_s$  is the duty cycle. There is unfortunately no simple closed-form expression for this summation. However, in the case the input is an ideally low-pass filtered white noise  $S_0$  of bandwidth  $B_n$ , the track part of the output PSD is given by

$$S_t(f) \cong D^2 \cdot S_0 \sum_{n=-N}^{+N} \text{sinc}^2(\pi n D), \quad (4.9)$$

where  $N$  is the closest integer to the undersampling factor  $N_{\text{under}} = B_n/(f_s/2)$ . For  $N \gg 1$  it can be shown that the above sum can be approximated by [1]

$$\sum_{n=-N}^{+N} \text{sinc}^2(\pi n D) \cong \sum_{n=-\infty}^{+\infty} \text{sinc}^2(\pi n D) = \frac{1}{D}. \quad (4.10)$$

This approximation can be extended to the case of a 1<sup>st</sup>-order low-pass filtered white noise having an input PSD given by

$$S_{in}(f) = \frac{S_0}{1 + (f/f_c)^2}. \quad (4.11)$$

For  $f_c T_s \gg 1$ , the track part of the output PSD can then be approximated by [1]

$$S_t(f) = D^2 \sum_{n=-\infty}^{+\infty} \text{sinc}^2(\pi n D) \cdot S_{in}\left(f - \frac{n}{T_s}\right) \cong D \cdot S_{in}(f). \quad (4.12)$$

This means that the track part of the T&H PSD is simply equal to the input PSD weighted by the duty cycle  $D$ . The above approximation (4.12) is validated numerically in Figure 4.4 for various cut-off frequencies  $f_c$  and duty cycles  $D$ .

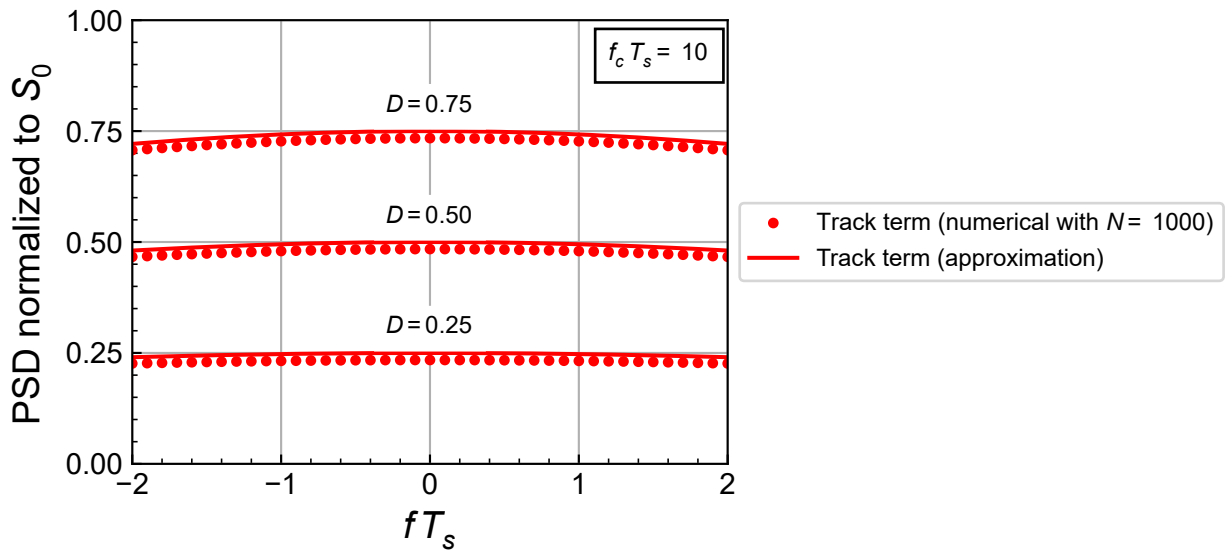


Figure 4.4: Numerical estimation of the track part of the output PSD compared to approximation (4.12).

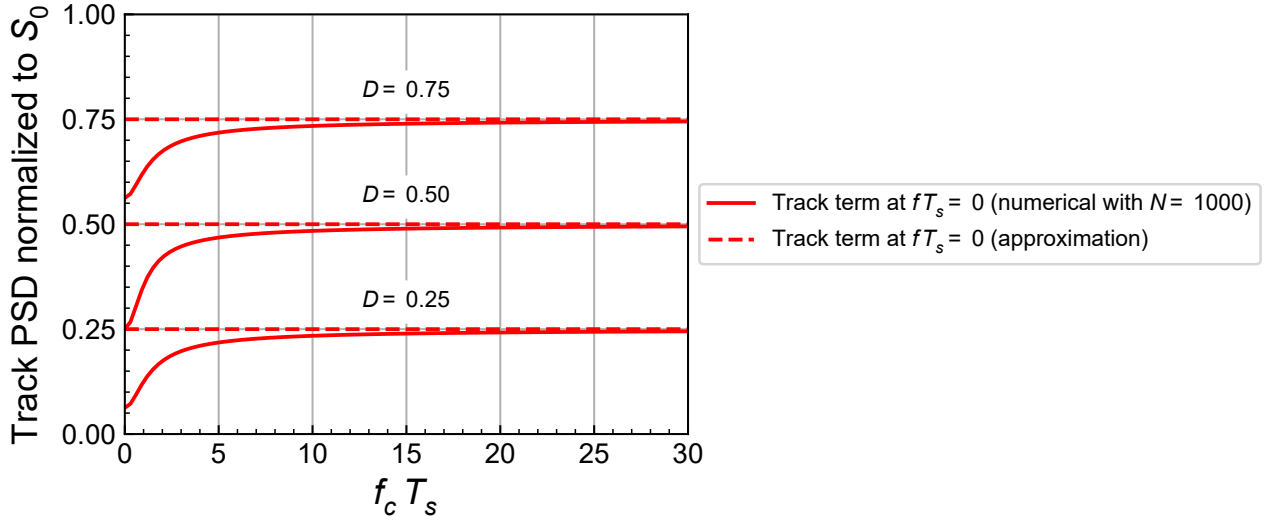


Figure 4.5: Track part of the output PSD versus bandwidth  $f_c T_s$  for different duty cycles  $D$ .

We see that the approximation slightly overestimates the numerical result for  $f_c T_s = 10$ . We can check that the numerical result converges to the approximation as  $f_c T_s$  tends to infinity. This is done in Figure 4.5.

We see from Figure 4.5 that the error becomes negligible for  $f_c T_s > 30$ .

The hold part of the output PSD  $S_h(f)$  can be calculated as shown above in the sample-and-hold section except that it is now weighted by the square of  $T_h/T_s = d = 1 - T_t/T_s = 1 - D$

$$\begin{aligned} S_h(f) &= \left(\frac{T_h}{T_s}\right)^2 \text{sinc}^2(\pi f T_h) \cdot \sum_{n=-\infty}^{+\infty} S_{in}\left(f - \frac{n}{T_s}\right) \\ &= d^2 \cdot \text{sinc}^2(\pi d f T_s) \cdot \sum_{n=-\infty}^{+\infty} S_{in}\left(f - \frac{n}{T_s}\right). \end{aligned} \quad (4.13)$$

In the case the input is a 1<sup>st</sup>-order low-pass filtered white noise, the hold part of the output PSD is given by

$$S_h(f) \cong \left(\frac{T_h}{T_s}\right)^2 \cdot \text{sinc}^2(\pi f T_h) \cdot \pi f_c T_s \cdot S_0 = d^2 \cdot \text{sinc}^2(\pi d f T_s) \cdot \pi f_c T_s \cdot S_0. \quad (4.14)$$

The track-and-hold output PSD can now be obtained by combining the track and the hold part resulting in

$$\begin{aligned} S_{out}(f) &= S_t(f) + S_h(f) \cong \frac{T_t}{T_s} \cdot S_{in}(f) + \left(\frac{T_h}{T_s}\right)^2 \cdot \text{sinc}^2(\pi f T_h) \cdot \pi f_c T_s \cdot S_0 \\ &= D \cdot S_{in}(f) + d^2 \cdot \text{sinc}^2(\pi d f T_s) \cdot \pi f_c T_s \cdot S_0 \\ &\cong \left[ D + d^2 \cdot \text{sinc}^2(\pi d f T_s) \cdot \pi f_c T_s \right] \cdot S_0. \end{aligned} \quad (4.15)$$

The approximation (4.15) is validated against the numerical computation in Figure 4.6.

We see that the approximation (4.15) gives an excellent estimation of the output PSD for a low-pass filtered white noise. We also notice that the noise PSD in the baseband is dominated by the hold part.

The increase of the white noise PSD in the baseband (actually at  $f = 0$ ) is given by

$$\xi \triangleq \frac{S_{out}(0)}{S_0} \cong D + d^2 \cdot \pi f_c T_s. \quad (4.16)$$

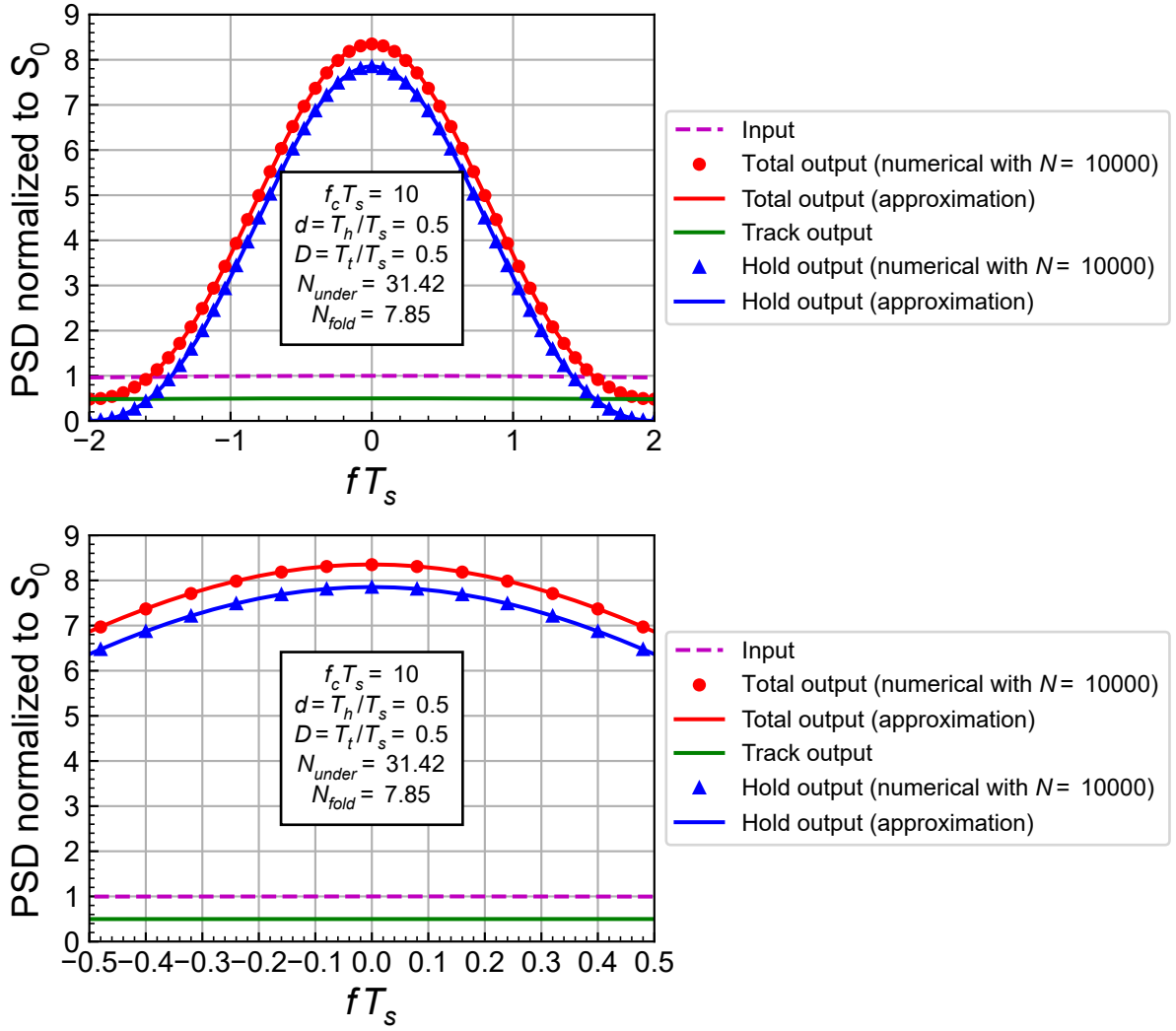


Figure 4.6: Comparison of the T&H output PSD calculated with the approximation (4.15) and numerically.

This relation is plotted versus the bandwidth  $f_c T_s$  for different duty cycles  $D$  in Figure 4.7 and versus the duty cycle  $D$  for different bandwidth  $f_c T_s$  in Figure 4.8.

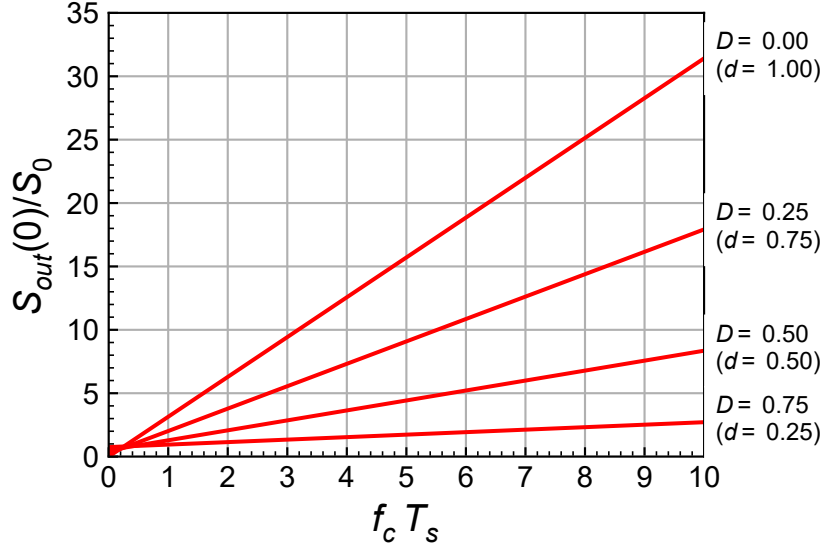


Figure 4.7: T&H PSD at  $f = 0$  versus bandwidth  $f_c T_s$  for different duty cycles  $D$ .

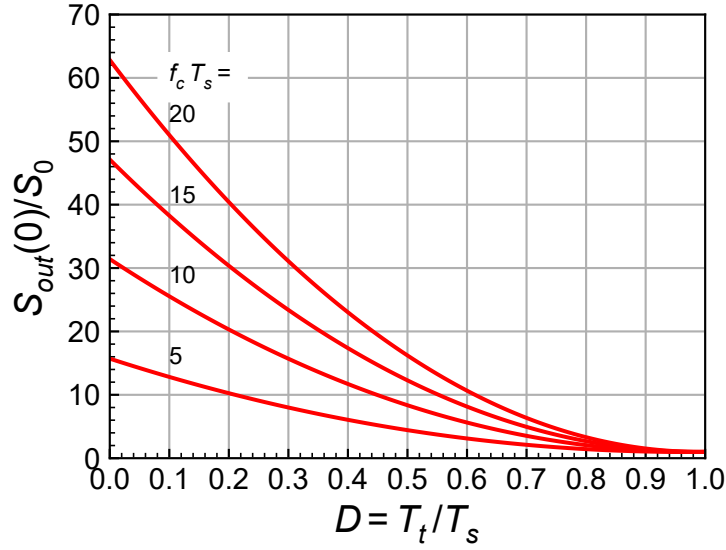


Figure 4.8: T&H PSD at  $f = 0$  versus duty cycles  $D$  for different bandwidth  $f_c T_s$ .

We see from Figure 4.8 that the noise folded back to the baseband decreases as the duty cycle  $D$  is increased. For a cut-off frequency  $f_c T_s = 10$ , it drops from about 30 in the case of the ideal sample-and-hold case to about 8 for the half duty cycle case.

We now will validate the above theory with Spectre simulations.

#### 4.2.2 Simulations

We will now validate the above result by Spectre simulations using the same 1<sup>st</sup>-order low-pass filtered white noise input and the parameters given in Table 4.1.

Table 4.1: Parameters used for the simulation of the white noise with Spectre.

| Parameter              | Value     | Unit          |
|------------------------|-----------|---------------|
| $f_s$                  | 10        | $kHz$         |
| $f_c$                  | 100       | $kHz$         |
| $f_k$                  | 1         | $kHz$         |
| $D \triangleq T_t/T_s$ | 0.5       | -             |
| $d \triangleq T_h/T_s$ | 0.5       | -             |
| $T_s$                  | 100       | $\mu s$       |
| $T_t$                  | 50        | $\mu s$       |
| $T_h$                  | 50        | $\mu s$       |
| $f_c T_s$              | 10        | -             |
| $f_k T_s$              | 0.1       | -             |
| $R_{lp}$               | 2         | $M\Omega$     |
| $S_0 = 4kTR_{lp}$      | 2.638e-14 | $V^2/Hz$      |
| $B_n$                  | 157       | $kHz$         |
| $V_n$                  | 64        | $\mu V_{rms}$ |

The simulated transfer function and input PSD are compared to the theoretical values in Figure 4.9. As expected we see a perfect match.

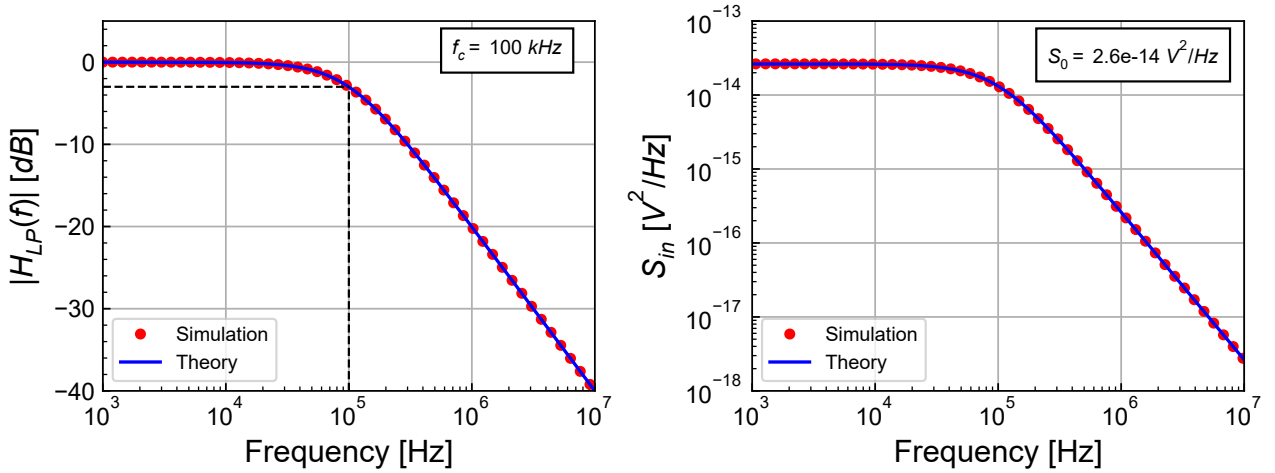


Figure 4.9: Spectre simulation of the transfer function and LP filtered white noise PSD.

Figure 4.10 shows the result of a transient noise analysis performed with Spectre using the parameters given in Table 4.1.

We clearly see that the input signal is tracked during the track period and sampled and held during the hold period. The output PSD is now simulated using Spectre with the `pss` and `pnoise` analysis. The result is shown in fig-th\_white\_noise\_psd\_sim.

To account for the non-overlapping phase we need to slightly adjust the duty cycle to a value smaller than  $1/2$ . With  $D = 0.49$ , we see a perfect match between the theoretical expression, the analytical approximation and the simulation results. Notice the shift of the zero of the  $\text{sinc}^2$  which is now at a frequency equal to  $fT_h = 1$  or  $f = 1/T_h = f_s/d \cong 2f_s = 20$  kHz. Notice also the track period contributes with a pedestal noise equal to  $S_0 \cdot d \cong S_0/2$ .

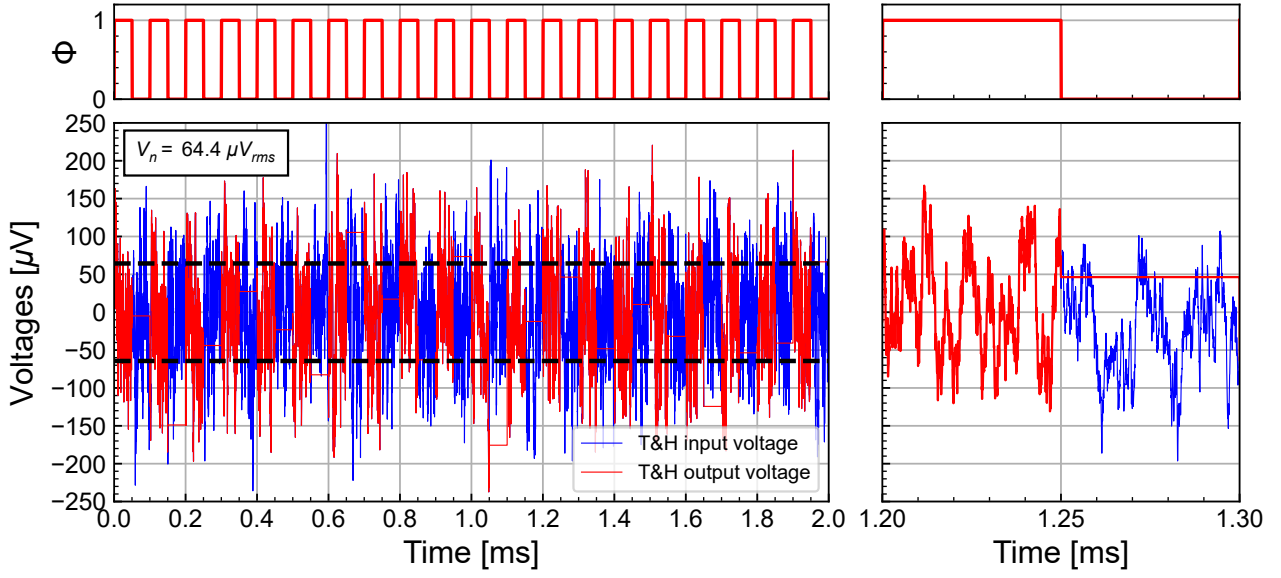


Figure 4.10: Spectre transient noise simulations showing the input and output waveforms.

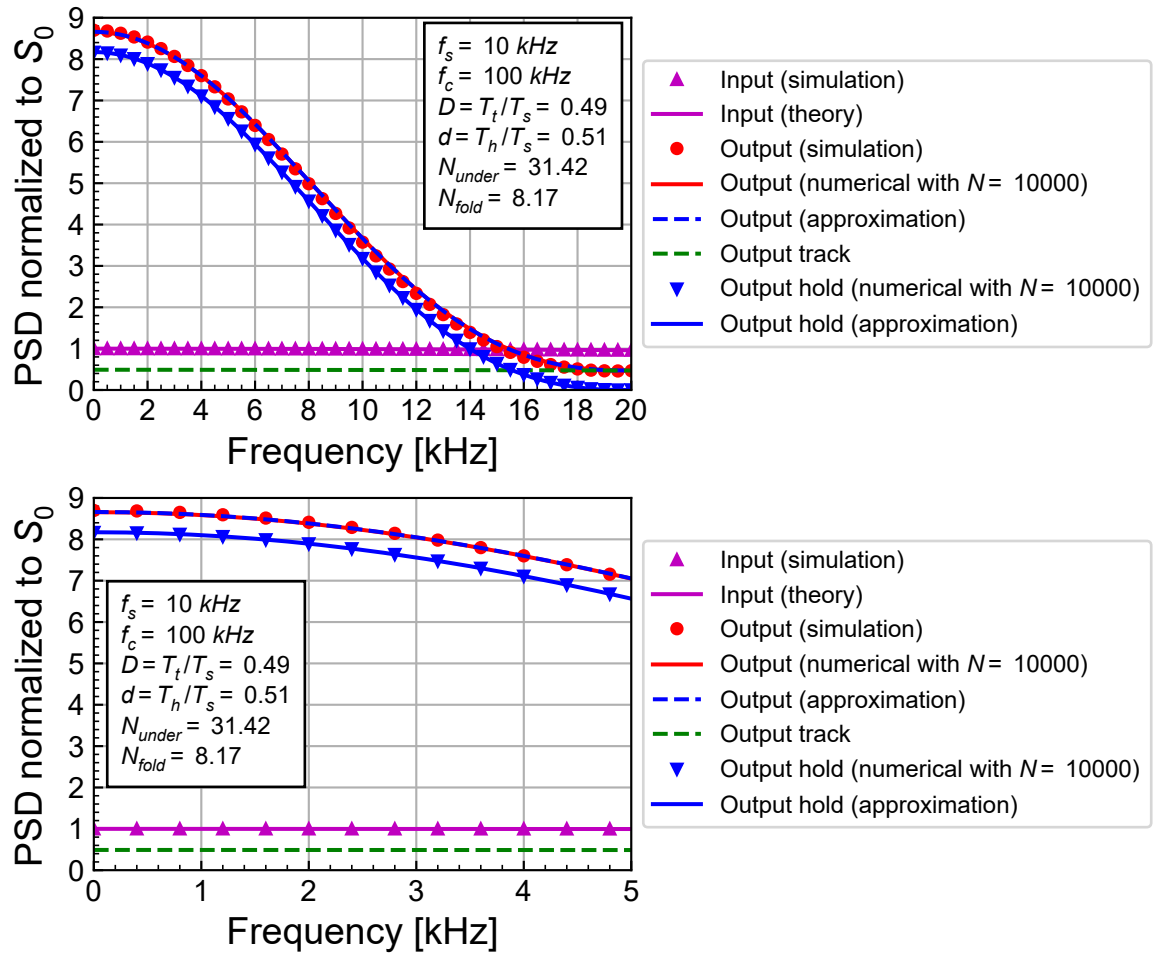


Figure 4.11: Spectre simulation of the output PSD for a LP filtered white noise input PSD.

## 4.3 Flicker noise sampling

### 4.3.1 Theory

For flicker noise, the input PSD is given by

$$S_{in}(f) = \frac{K_f}{|f|} \cdot \frac{1}{1 + (f/f_c)^2} = \frac{K_f T_s}{|f T_s|} \cdot \frac{1}{1 + \left(\frac{f T_s}{f_c T_s}\right)^2}. \quad (4.17)$$

In the case of flicker noise we need to account for the correlation existing between the track and the hold period occuring for the baseband signals. This correlation is key to noise reduction techniques such as autozero or correlated double sampling (CDS) which take advantage of the correlation to actually reduce the flicker noise [9]. On the other hand, we can assume that the other frequency components that are aliased from high frequency into the baseband can be considered as uncorrelated. This means that we need to split the track-and-hold output PSD into a baseband part  $S_{bb}(f)$  and an uncorrelated foldover part  $S_{fold}(f)$  according to

$$S_{out}(f) = S_{bb}(f) + S_{fold}(f), \quad (4.18)$$

where the baseband PSD  $S_{bb}(f)$  is given by

$$S_{bb}(f) = [D + d \cdot \text{sinc}(\pi d f T_s)]^2 \cdot S_{in}(f) = [1 - d + d \cdot \text{sinc}(\pi d f T_s)]^2 \cdot S_{in}(f). \quad (4.19)$$

For sufficiently large duty cycle  $D$  (small  $d$ ), the first zero of the  $\text{sinc}(\pi d f T_s)$  function appears at  $1/(dT_s)$  which is higher than the sampling frequency. Hence, in the baseband we can consider that

$$\text{sinc}(\pi d f T_s) \cong 1 \quad (4.20)$$

and therefore

$$S_{bb}(f) \cong S_{in}(f). \quad (4.21)$$

This means that for sufficiently large duty cycles, the baseband output PSD is almost equal to the input PSD. The baseband PSD is compared to the input PSD in Figure 4.12 for  $f_c T_s = 10$  and  $D = d = 0.5$ .

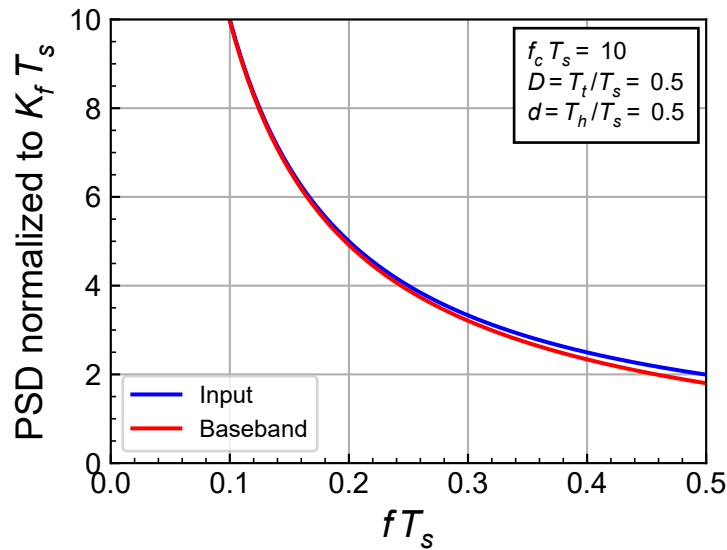


Figure 4.12: T&H baseband component compared to input PSD for  $f_c T_s = 10$  and  $D = d = 0.5$ .

As expected, we see that the baseband component is almost equal to the input PSD. This explains why flicker noise can be reduced by subtracting the sample-and-held noise to the instantaneous noise [9].

The foldover term includes aliased components from both, the track and the hold signals

$$S_{fold}(f) = S_{t,fold}(f) + S_{h,fold}(f). \quad (4.22)$$

The foldover coming from the track signal is given by

$$\begin{aligned} S_{t,fold}(f) &= \left(\frac{T_t}{T_s}\right)^2 \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \text{sinc}^2\left(n\pi \frac{T_t}{T_s}\right) \cdot S_{in}\left(f - \frac{n}{T_s}\right) \\ &= D^2 \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \text{sinc}^2(n\pi D) \cdot S_{in}(f T_s - n). \end{aligned} \quad (4.23)$$

There is unfortunately no simple closed form expression for this summation in case of the low-pass filtered flicker noise PSD. Nevertheless, we can try to find an approximation by first assuming an infinite bandwidth  $f_c T_s \gg 1$ . The input PSD then simplifies to

$$S_{in}(f) \cong \frac{K_f}{|f|} = \frac{K_f T_s}{|f T_s|} \quad (4.24)$$

It can be shown that for  $D = d = 0.5$ , the track foldover component can be approximated by its value at  $f = 0$  which is given by

$$\begin{aligned} S_{t,fold}(f) &\cong S_{t,fold}(0) = D^2 \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \text{sinc}^2\left(n\frac{\pi}{2}\right) \cdot \frac{K_f T_s}{|n|} \\ &= D^2 \cdot K_f T_s \cdot \frac{7}{\pi^2} \zeta(3) \cong 0.852557 D^2 \cdot K_f T_s. \end{aligned} \quad (4.25)$$

where  $\zeta(s)$  is the [Riemann zeta function](#). The above approximation is validated with numerical computation in Figure 4.13.

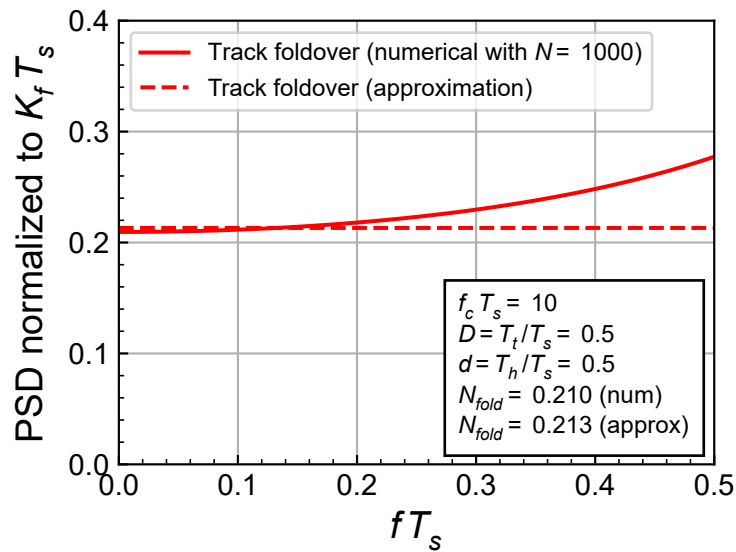


Figure 4.13: Approximation of track foldover component for flicker noise.

We see that the above approximation gives a good estimation of the track foldover at low frequency.

The dependence of the track foldover PSD to the duty cycle  $D$  can be approximated by

$$S_{t,fold} \cong D^2 \cdot 1.27 (|\ln(D)|)^{1.4} \cdot K_f T_s. \quad (4.26)$$

This approximation is plotted versus the duty cycle  $D$  and compared to the result of the numerical computation in Figure 4.14.

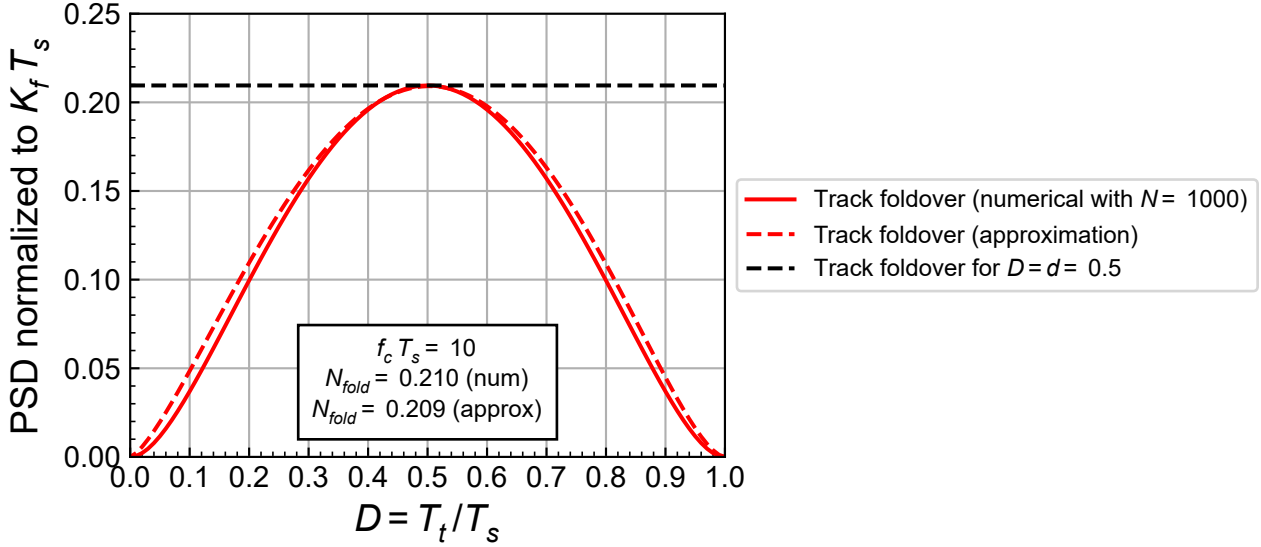


Figure 4.14: T&H flicker noise track foldover versus duty cycle  $D$ .

We observe that the track foldover is maximum for half duty cycle ( $D = d = 0.5$ ), while it vanishes for  $D = 0$  and  $D = 1$ . Indeed for  $D = 0$  there is no track but only hold foldover, whereas for  $D = 1$  there is no hold and no track foldover because there is no more sampling since the circuit operates in continuous time.

The foldover term coming from the hold signal is the same as the one we already have calculated for the sample-and-hold signal except that it needs to be multiplied by  $(T_h/T_s)^2 = d^2$

$$\begin{aligned} S_{h,fold}(f) &= \left(\frac{T_h}{T_s}\right)^2 \cdot \text{sinc}^2(\pi f T_h) \cdot \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} S_{in}\left(f - \frac{n}{T_s}\right) \\ &= d^2 \cdot \text{sinc}^2(\pi d f T_s) \cdot \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} S_{in}(f T_s - n). \end{aligned} \quad (4.27)$$

We can reuse the approximation used above for the sample-and-hold case resulting in

$$S_{h,fold}(f T_s) \cong d^2 \cdot \text{sinc}^2(\pi d f T_s) \cdot K_f T_s \cdot 2 [\gamma + \ln(f_c T_s)], \quad (4.28)$$

where  $\gamma \cong 0.577216$  is the Euler constant. The above approximation is compared to the result obtained from numerical computations in Figure 4.15.

We see a very good agreement between the constant approximation and the result of the numerical computation.

The hold foldover component is plotted versus the duty cycle  $D$  in Figure 4.16.

We see that the hold foldover is maximum for  $D = 0$  ( $d = 1$ ) (sample-and-hold case) and decreases with  $D$  to reach zero when  $D = 1$  because there is no sampling anymore. For  $D = d = 0.5$ , the hold foldover normalized to  $K_f T_s$  is equal to  $N_{fold} = 1.440$ .

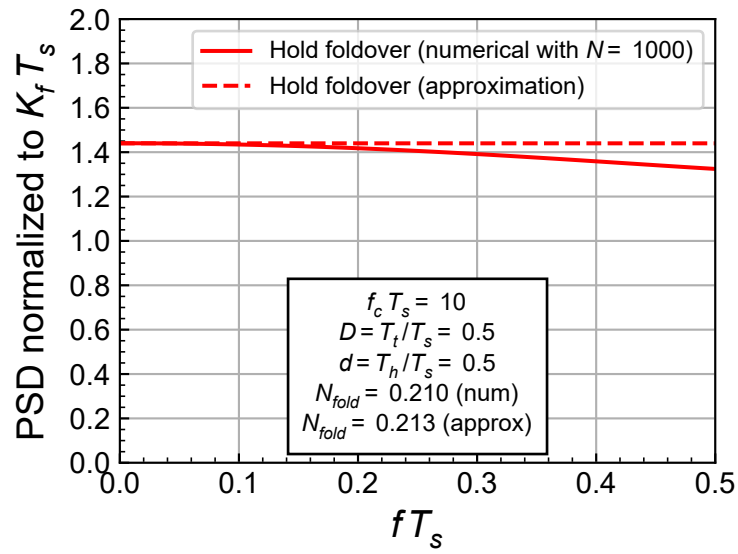
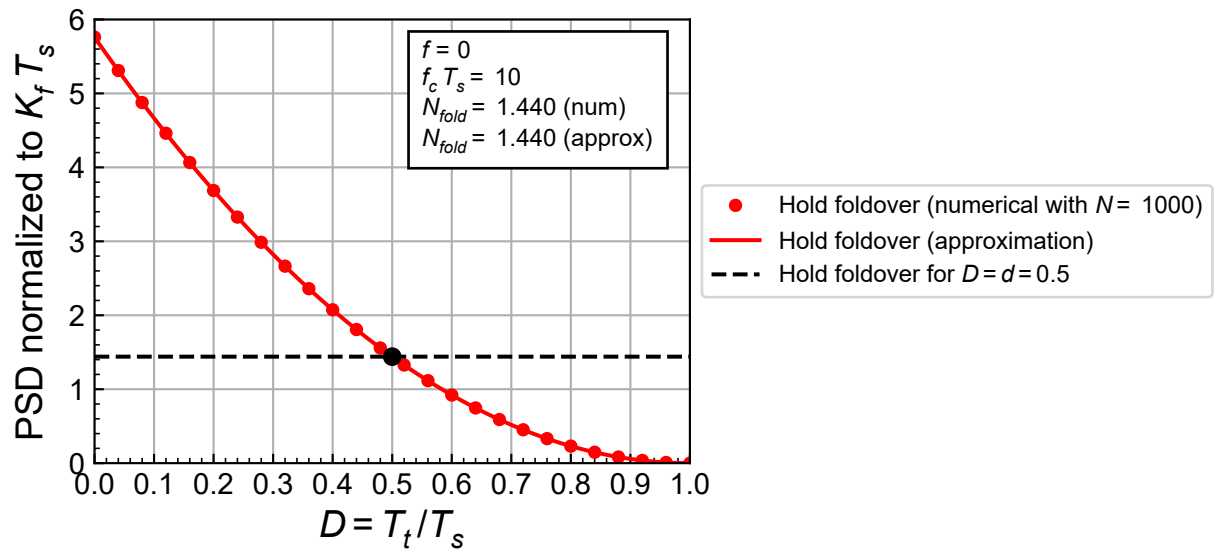


Figure 4.15: T&amp;H hold foldover component approximation for flicker noise.

Figure 4.16: T&H hold foldover component versus duty cycle  $D$ .

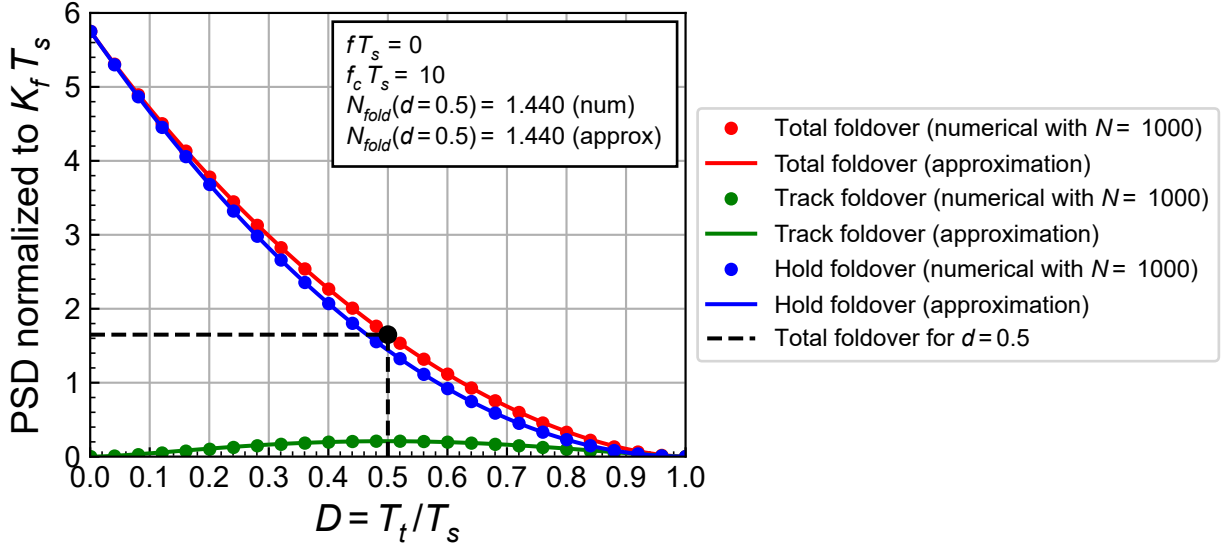


Figure 4.17: T&amp;H flicker noise foldover components.

The foldover components due to the track and the hold signals are plotted versus the duty cycle  $D$  in Figure 4.17.

We see that the contribution of the track signal to the foldover PSD is actually negligible. Hence we can basically ignore the track contribution for the flicker noise foldover.

The track, hold and total foldover PSD are plotted versus frequency in the baseband in Figure 4.18.

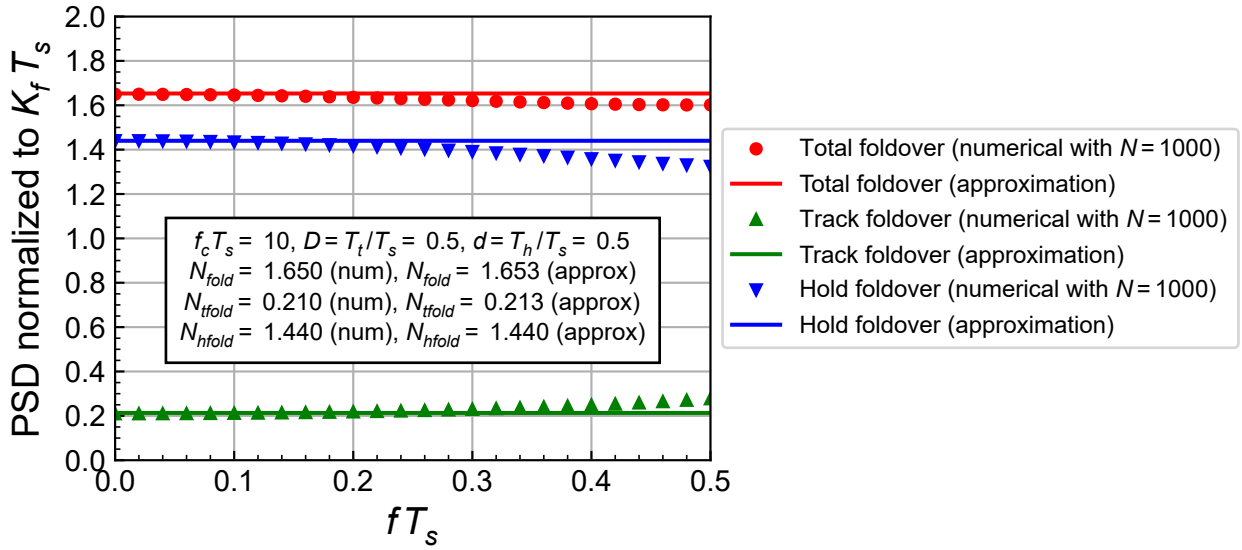


Figure 4.18: Track, hold and total foldover PSD for flicker noise.

The total flicker noise foldover PSD can then be approximated by

$$S_{fold}(f) \cong K_f T_s \cdot \left[ D^2 \cdot 1.27 (|\ln(D)|)^{1.4} + d^2 \cdot \text{sinc}^2(\pi d f T_s) \cdot 2(\gamma + \ln(f_c T_s)) \right] \quad (4.29)$$

Finally, the simple approximation of the output PSD is compared to numerical computation in the baseband in Figure 4.19.

We see an almost perfect match between the simple approximation and the numerical result computed for  $N$  terms in the summation.

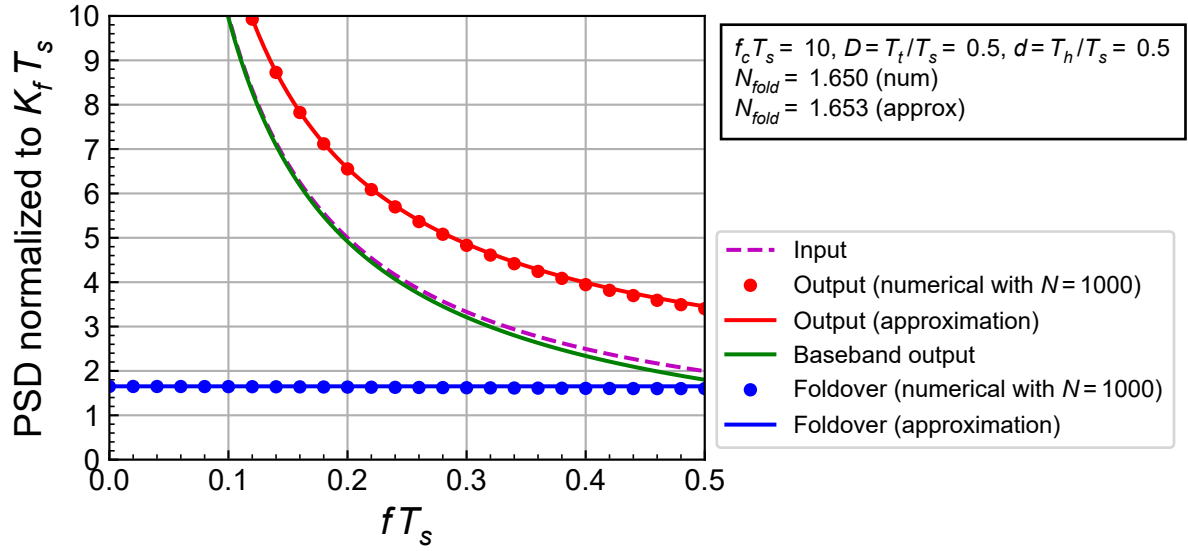


Figure 4.19: T&amp;H output PSD

The flicker noise tT&H model is validated with Spectre simulations in the next section.

#### 4.3.2 Simulations

We now will compare the above T&H model for flicker noise to the results of Spectre simulations using the parameters given in Table 4.2.

Table 4.2: Parameters used for flicker noise Spectre simulation.

| Parameter              | Value     | Unit          |
|------------------------|-----------|---------------|
| $f_s$                  | 10        | $kHz$         |
| $f_c$                  | 100       | $kHz$         |
| $f_k$                  | 1         | $kHz$         |
| $D \triangleq T_t/T_s$ | 0.5       | -             |
| $d \triangleq T_h/T_s$ | 0.5       | -             |
| $T_s$                  | 100       | $\mu s$       |
| $T_t$                  | 50        | $\mu s$       |
| $T_h$                  | 50        | $\mu s$       |
| $f_c T_s$              | 10        | -             |
| $f_k T_s$              | 0         | -             |
| $R_{lp}$               | 2         | $M\Omega$     |
| $S_0 = 4kTR_{lp}$      | 2.638e-14 | $V^2/Hz$      |
| $B_n$                  | 157       | $kHz$         |
| $V_n$                  | 64        | $\mu V_{rms}$ |
| $K_f$                  | 2.638e-11 | $V^2$         |
| $K_f T_s$              | 2.638e-15 | $V^2/Hz$      |
| $S_{fold}(0)$          | 7.962e-13 | $V^2/Hz$      |
| $(T_h/T_s)^2$          | 0.0001    | -             |

The transfer function and input PSD are shown in Figure 4.20. As expected the simulation results perfectly correspond to the theoretical model.



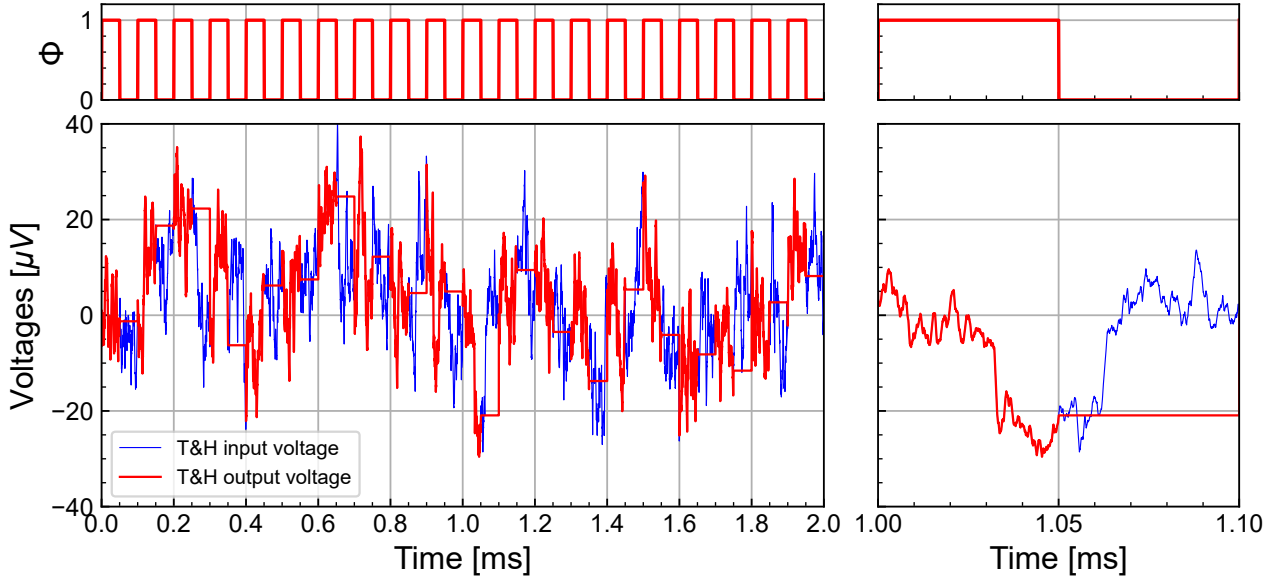


Figure 4.21: Spectre transient noise simulations showing the input and output waveforms for a flicker noise input.

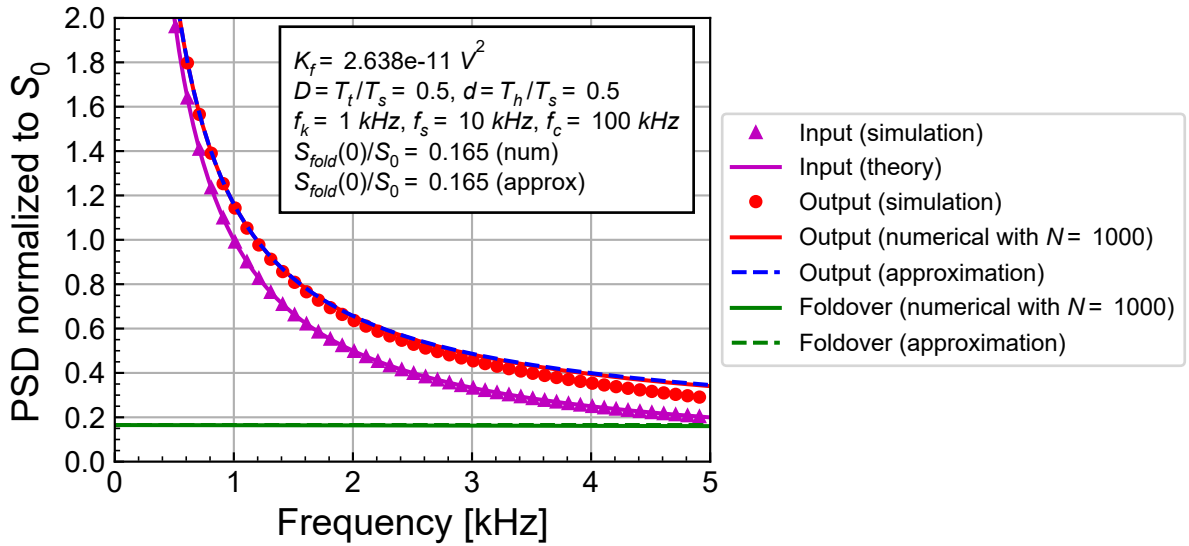


Figure 4.22: Spectre simulation of the output PSD for a LP filtered flicker noise input PSD.

We see a good fit between the approximation and the numerical computation and simulation result. The output PSD is almost equal to the inputs PSD with a small constant increase due to the foldover term.

We can extract the foldover component from the simulation as

$$S_{fold,sim}(f) = S_{out,sim}(f) - S_{bb}(f) \quad (4.30)$$

assuming that the baseband component is given by

$$S_{bb}(f) = [D + (1 - D) \cdot \text{sinc}(\pi(1 - D)f T_s)]^2 \cdot S_{in,sim}(f). \quad (4.31)$$

The extracted foldover component is then compared to the result of the numerical computation and the approximation below.

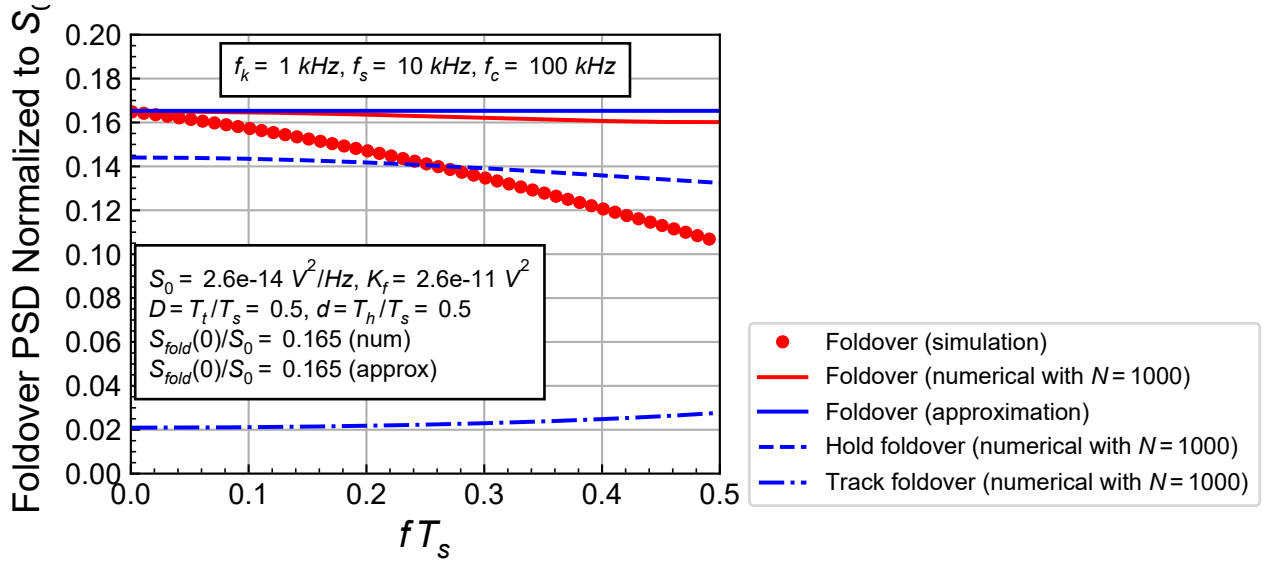


Figure 4.23: T&H foldover component extracted from the sSpectre simulation and compared to the model.

## 4.4 Combined white and flicker noise sampling

### 4.4.1 Theory

We can now combine the above result to model the track-and-hold process of a noise including both white and flicker noise. As in the sample-and-hold section, the PSD of the noise source can then be written as

$$S_n(f) = S_0 \cdot \left(1 + \frac{f_k}{|f|}\right), \quad (4.32)$$

where  $S_0$  is the white noise PSD and  $f_k$  is the corner frequency. The input PSD of the track-and-hold circuit is then given by

$$S_{in}(f) = \frac{S_n(f)}{1 + (f/f_c)^2} = \frac{S_0}{1 + (f/f_c)^2} \cdot \left(1 + \frac{f_k}{|f|}\right). \quad (4.33)$$

To calculate the output PSD it is better to split the part coming from the white noise and that coming from the flicker noise

$$S_{out}(f) = S_{out,white}(f) + S_{out,flicker}(f). \quad (4.34)$$

The output PSD due to the white noise is then given by

$$\begin{aligned} S_{out,white}(f) &\cong D \cdot S_{in}(f) + d^2 \cdot \text{sinc}^2(\pi d f T_s) \cdot \pi f_c T_s \cdot S_0 \\ &\cong \left[ D + d^2 \cdot \text{sinc}^2(\pi d f T_s) \cdot \pi f_c T_s \right] \cdot S_0. \end{aligned} \quad (4.35)$$

The output PSD due to the flicker noise part is given by

$$S_{out,flicker}(f) = S_{bb,flicker}(f) + S_{fold,flicker}(f) \quad (4.36)$$

where  $S_{bb,flicker}(f)$  is the flicker noise baseband component

$$S_{bb,flicker}(f) = [D + d \cdot \text{sinc}(\pi d f T_s)]^2 \cdot S_{in}(f) \cong S_{in,flicker}(f), \quad (4.37)$$

with

$$S_{in,flicker}(f) = \frac{S_0 f_k}{|f|} \cdot \frac{1}{1 + (f/f_c)^2} \quad (4.38)$$

is the flicker noise part of the input PSD.

The flicker noise foldover PSD can be approximated by

$$\begin{aligned} S_{fold,flicker}(f) &\cong K_f T_s \cdot \left[ D^2 \cdot 1.27 (|\ln(D)|)^{1.4} + d^2 \cdot \text{sinc}^2(\pi d f T_s) \cdot 2(\gamma + \ln(f_c T_s)) \right] \\ &\cong K_f T_s \cdot d^2 \cdot \text{sinc}^2(\pi d f T_s) \cdot 2(\gamma + \ln(f_c T_s)) \end{aligned} \quad (4.39)$$

where  $\gamma \cong 0.577216$  is the Euler constant.

The above approximation will now be compared to simulations using Spectre.

#### 4.4.2 Simulations

The above expressions of the output PSD will now be validated by Spectre simulations using the parameters given in Table 4.3.

Table 4.3: Parameters used for the Spectre simulation of the T&H with combined input.

| Parameter              | Value     | Unit          |
|------------------------|-----------|---------------|
| $f_s$                  | 10        | $kHz$         |
| $f_c$                  | 100       | $kHz$         |
| $f_k$                  | 1         | $kHz$         |
| $D \triangleq T_t/T_s$ | 0.5       | -             |
| $d \triangleq T_h/T_s$ | 0.5       | -             |
| $T_s$                  | 100       | $\mu s$       |
| $T_t$                  | 50        | $\mu s$       |
| $T_h$                  | 50        | $\mu s$       |
| $f_c T_s$              | 10        | -             |
| $f_k T_s$              | 0         | -             |
| $R_{lp}$               | 2         | $M\Omega$     |
| $S_0 = 4kTR_{lp}$      | 2.638e-14 | $V^2/Hz$      |
| $B_n$                  | 157       | $kHz$         |
| $V_n$                  | 64        | $\mu V_{rms}$ |
| $K_f$                  | 2.638e-11 | $V^2$         |
| $K_f T_s$              | 2.638e-15 | $V^2/Hz$      |

The simulated transfer function and input PSD are shown in Figure 4.24. As expected, the shape of the transfer function and input PSD correspond to the theoretical expressions.

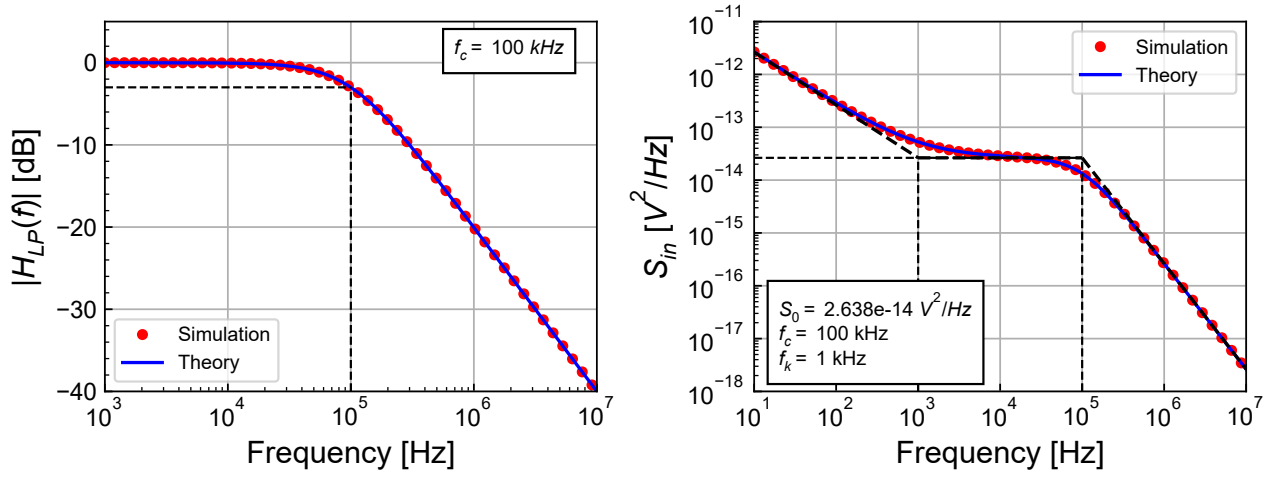


Figure 4.24: Spectre simulation of the transfer function and LP filtered combined input noise PSD.

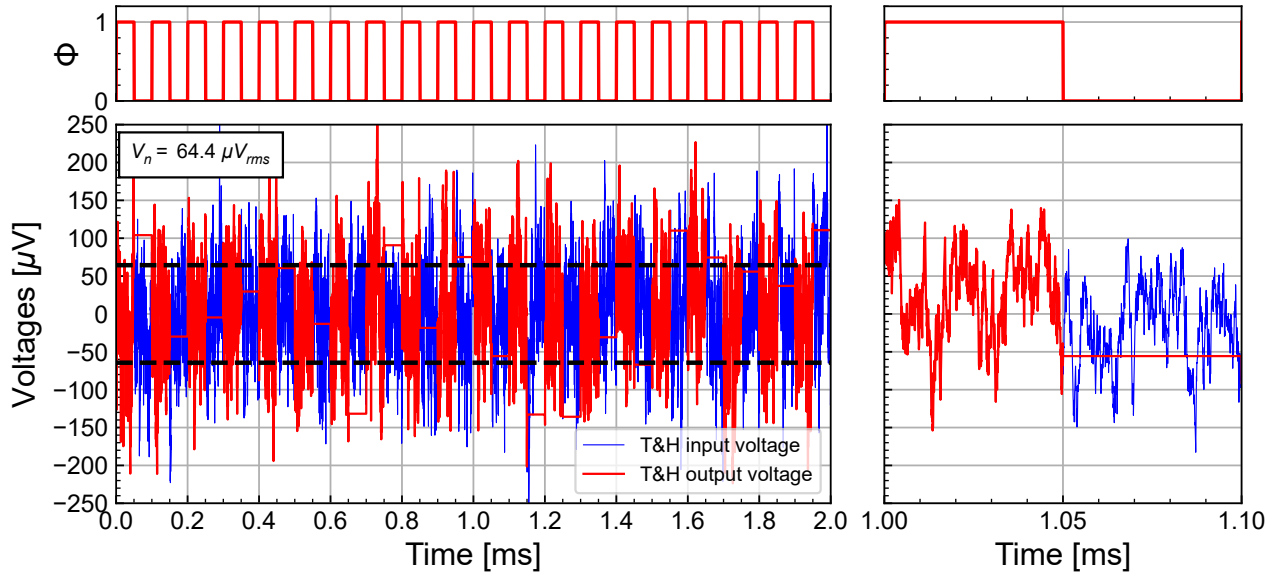


Figure 4.25: Spectre transient noise simulations showing the input and output waveforms for a combined noise input.

Figure 4.25 shows a Spectre transient noise simulation with the input waveform in blue and the T&H output waveform in red.

The simulated output PSD is compared to the numerical expression and approximation in Figure 4.26. We see a very good agreement between the simulations, the numerical calculation and the analytic approximation.

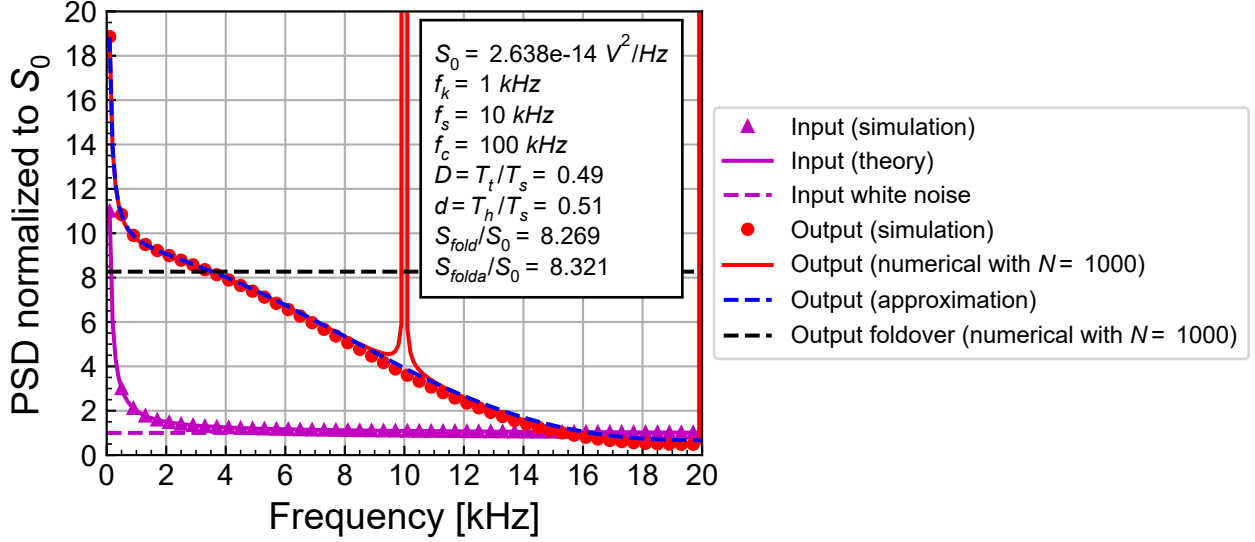


Figure 4.26: Spectre simulation of the output PSD for a LP filtered flicker noise input PSD.

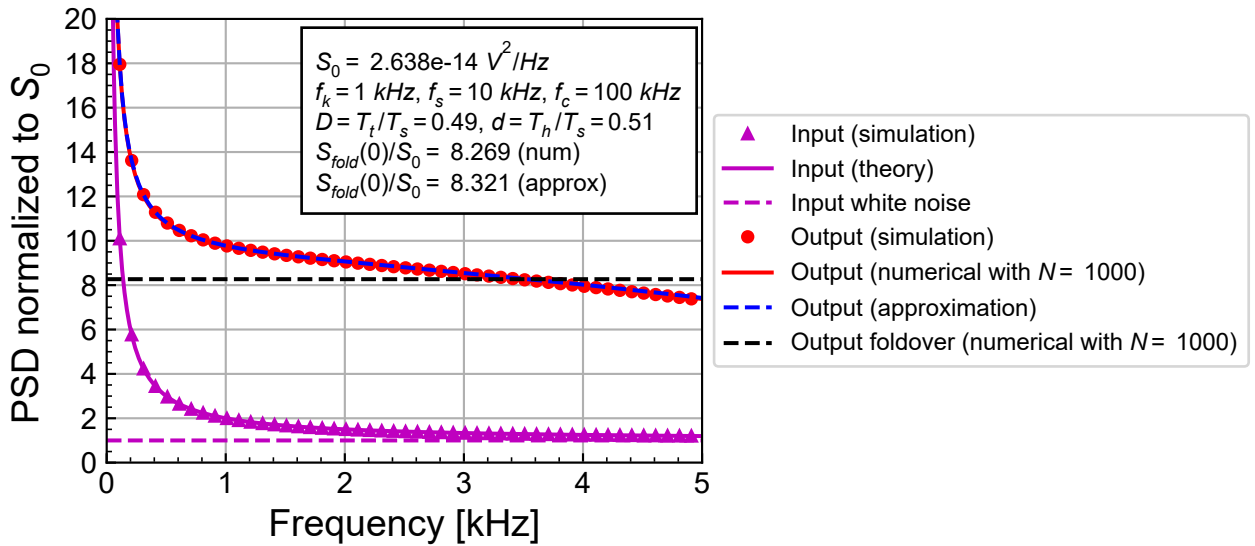


Figure 4.27: Spectre simulation of the output PSD for a LP filtered combined noise input PSD in the baseband.

Figure 4.27 shows a detailed view of the simulation, the numerical calculation and the analytic approximation in the baseband. We see an almost perfect match between the simulation and the numerical and approximate models. It clearly illustrates the increase of the white noise in the baseband by a factor  $S_{fold}(0)/S_0 \cong 8.3$  due to the foldover components of both white and flicker noise.

The same data is plotted with a log x-axis in Figure 4.28.

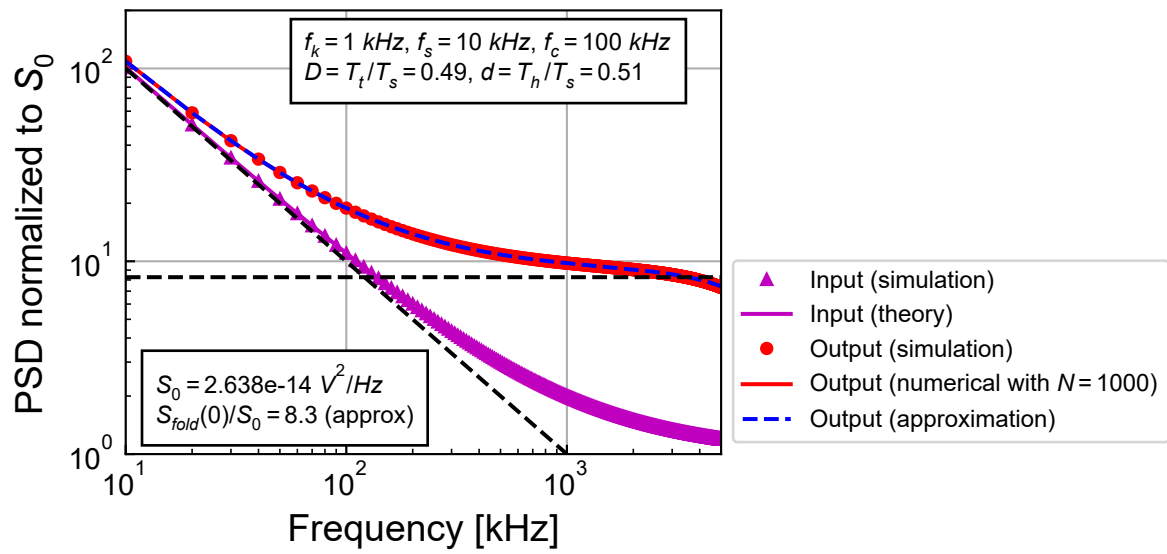


Figure 4.28: Spectre simulation of the output PSD for a LP filtered combined noise input PSD in the baseband.

## 5 Conclusion

In this notebook we have derived simple models for the estimation of the sampled noise PSD for both the ideal sample-and-hold circuit and the track-and-hold case.

In the case of the ideal sample-and-hold, the noise in the baseband is increased by a foldover component which comes from the high frequency noise that is aliased back to the baseband due to sampling. The white noise is increased by the undersampling factor defined as the ratio of the noise bandwidth to the Nyquist frequency  $f_s/2$ . For a 1<sup>st</sup>-order low-pass filtered white noise having a cut-off frequency  $f_c$ , the undersampling factor is  $N_{under} = \pi f_c T_s$  which is usually much larger than one since the settling time needs to be made much smaller than the sampling period  $T_s$ . The foldover also includes the queues of the aliased flicker noise PSD which can be approximated by a white noise in the baseband with a PSD equal to  $2K_f T_s \cdot [\gamma + \ln(f_c T_s)]$ . The total foldover component that adds to the input noise is then  $S_0 \cdot \{\pi f_c T_s - 1 + 2f_k T_s \cdot [\gamma + \ln(f_c T_s)]\}$  where  $f_k$  is the corner frequency. This simple model has been successfully validated numerically and compared to the results of simulation using Cadence Spectre. The impact of the ideal sample-and-hold can be dramatic since it increases the baseband noise significantly. It can be shown that actually all the noise power initially distributed over a wide frequency range in the input PSD is down-converted to the Nyquist band resulting in an increased of the PSD to maintain the noise power constant.

The track-and-hold case was also analyzed accounting for the possible correlation existing between the track and the hold period. The output signal is decomposed into a track and a hold signal. In the case of a 1<sup>st</sup>-order low-pass filtered white noise at the input, the correlation between the track and the hold period can be ignored. The track output PSD is then simply equal to the input PSD weighted by the duty cycle  $d = T_t/T_s$ . The hold PSD is equal to what was derived for the ideal sample-and-hold case except that it is weighted by  $(T_h/T_s)^2 = (1 - d)^2$ . The impact of aliasing is therefore mitigated by the duty cycle. Indeed, for a cut-off frequency  $f_c$  equal to ten times the sampling frequency, the undersampling factor of the ideal sample-and-hold  $N_{under} = 31.4$  is reduced to  $N_{under} = 7.85$  for a duty cycle  $d = 0.5$ . For the flicker noise, it is crucial to account for the correlation existing between the track and the hold period. Indeed, it is really thanks to this correlation that low-frequency noise reduction techniques such autozero or correlated-double-sampling (CDS) can be performed. However the correlation can be ignored for the aliased components. The signal is therefore decomposed into a baseband component, which is almost equal to the input PSD and a foldover term. The simple analytical model has been validated by numerical results and by simulations using Spectre. Finally, the model of the track-and-hold for a noise including both a white and a flicker noise PSD has been validated with numerical computations and with Spectre simulations.

## References

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