Definition of the Saturation Voltage

Using the EKV Model (Version 1)

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This notebook presents different definitions of the saturation voltage of a MOS transistor that is valid in all modes of inversion, from weak to strong inversion. It then ends with a simple approximation of the normalized drain-to-source saturation voltage that can be used for design.

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1 Introduction

This notebook presents different definitions of the saturation voltage of a MOS transistor that is valid in all modes of inversion, from weak to strong inversion. It then ends with a simple approximation of the normalized drain-to-source saturation voltage that can be used for design.

2 Definition of the saturation voltage

The saturation voltage is the drain-to-source voltage V_{DS} above which the drain current saturates to the foward current I_F [1]. In strong inversion, the saturation voltage corresponds to the pinch-off voltage V_P and hence the drain-to-source saturation voltage is given by $V_{DSsat} = V_P - V_S$ [1]. In weak inversion, we cannot defined a drain-to-source saturation voltage because the drain current tends asymptotically to I_F. We cannot use $V_P - V_S$ because it becomes zero in moderate inversion or negative in weak inversion. In order to try defining a saturation voltage in weak inversion, we rewrite the drain current in weak inversion as [1]

$$I_D = I_F \left[1 - e^{-\frac{V_{DS}}{U_T}}\right] \tag{2.1}$$

The relative error between the actual drain current I_D and the asymptotic value I_F is defined as [1]

$$\varepsilon \triangleq \left|\frac{I_D - I_F}{I_F}\right| = e^{-\frac{V_{DS}}{U_T}} = e^{-v_{ds}}. \tag{2.2}$$

The relative error ε decreases exponentially with $v_{ds} = V_{DS}/U_T$. The saturation voltage in weak inversion can be defined as the drain-to-source voltage for a given error ε. Taking a saturation voltage of 4 U_T, corresponding to $V_{DSsat} = 103\ mV$ at room temperature ($T = 26.9\ °C$), gives and error $\varepsilon = 1.832\ \%$.

The drain current can unfortunately not be expressed directly in term of the V_{DS} voltage. However, we can express the voltages versus the forward and reverse currents. Indeed, the voltages are related to the normalized charges at the source and drain according to [1] [2] [3]

$$v_p - v_s = \ln(q_s) + 2q_s, \tag{2.3}$$
$$v_p - v_d = \ln(q_d) + 2q_d. \tag{2.4}$$

From (2.3) and (2.4) we can express the normalized drain-to-source voltage v_{ds} as a function of the normalized charges at the source q_s and drain q_d as

$$v_{ds} = v_d - v_s = 2(q_s - q_d) + \ln\left(\frac{q_s}{q_d}\right). \tag{2.5}$$

Recalling that the normalized charges depend on the normalized forward and reverse currents according to

$$q_s = \frac{\sqrt{4i_f + 1} - 1}{2}, \tag{2.6}$$
$$q_d = \frac{\sqrt{4i_r + 1} - 1}{2}, \tag{2.7}$$

we can express v_{ds} in terms of i_f and i_r. We can plot $i_d/i_f = (i_f - i_r)/i_f$ versus the v_{ds} voltage in Figure 2.1 for different inversion coefficient IC.

Figure 2.1 illustrates how the saturation voltage decreases as we move progressively from strong inversion to moderate and weak inversion.

We can rewrite (2.5) in terms of the q_d/q_s ratio as

$$v_{ds} = 2q_s\left(1 - \frac{q_d}{q_s}\right) - \ln\left(\frac{q_d}{q_s}\right). \tag{2.8}$$

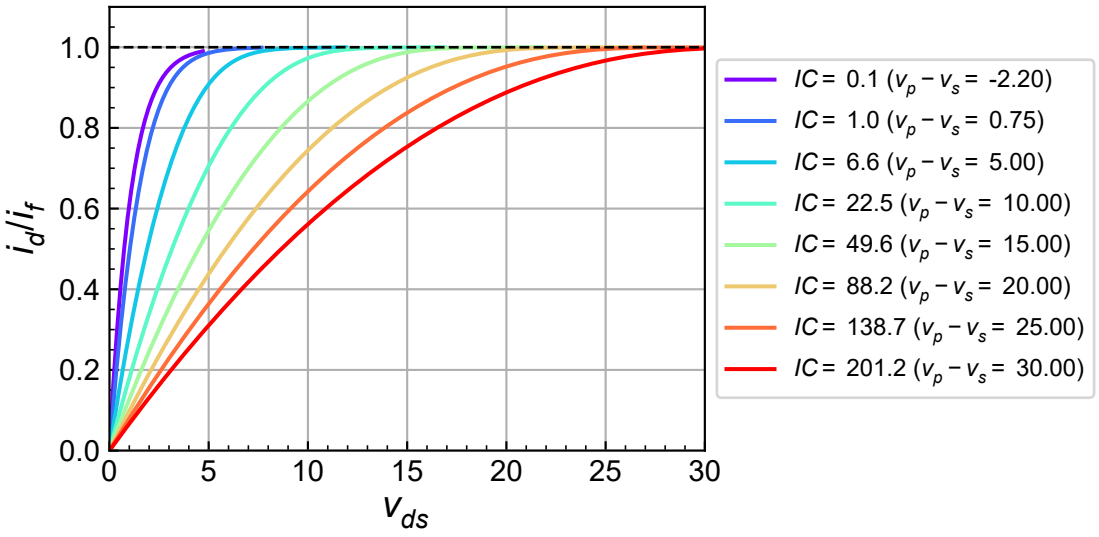

Figure 2.1: Normalized drain current i_d/i_f versus normalized drain-to-source voltage v_{ds}.

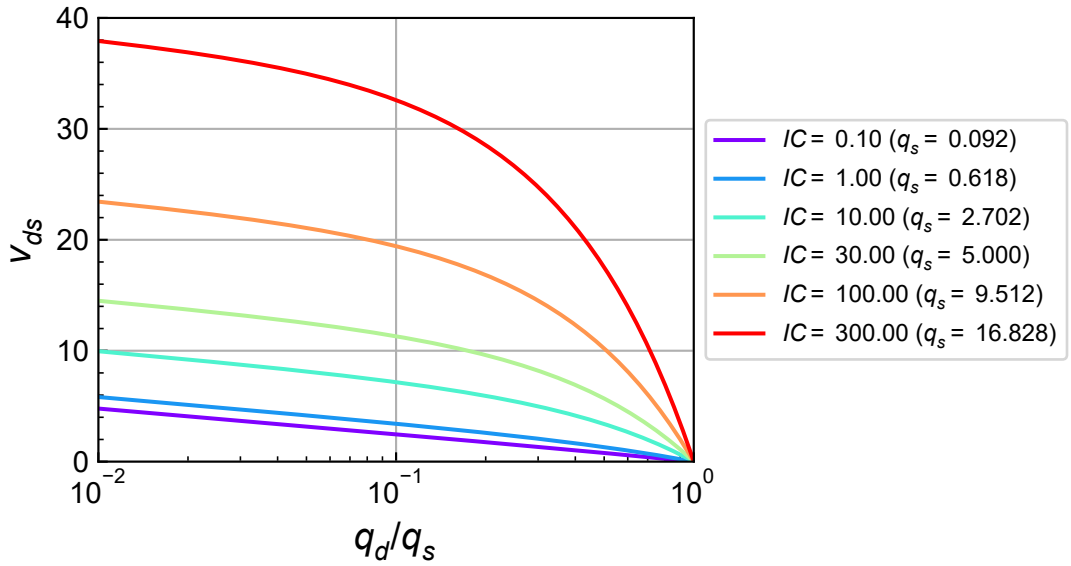

Figure 2.2: v_{ds} voltage versus the q_d/q_s ratio for various IC.

Equation (2.8) is plotted in Figure 2.2 for different values of IC and hence for different q_s. We see that for $q_d/q_s = 1$ we are in the linear region and $v_{ds} = 0$. Decreasing the q_d/q_s ratio moves the operating point from the linear region into the saturation region increasing the v_{ds} voltage.

We could therefore also use the q_d/q_s ratio (or its inverse the q_s/q_d ratio) as a criterium to define the saturation voltage

$$v_{dssat} = 2q_s\,(1-r) - \ln(r), \tag{2.9}$$

where $r = q_d/q_s$ and q_s depends on the inversion coefficient according to

$$q_s = \frac{\sqrt{4IC+1}-1}{2}. \tag{2.10}$$

Equation (2.9) is plotted version the inversion coefficient in Figure 3.3 for various values of $r = q_d/q_s$.

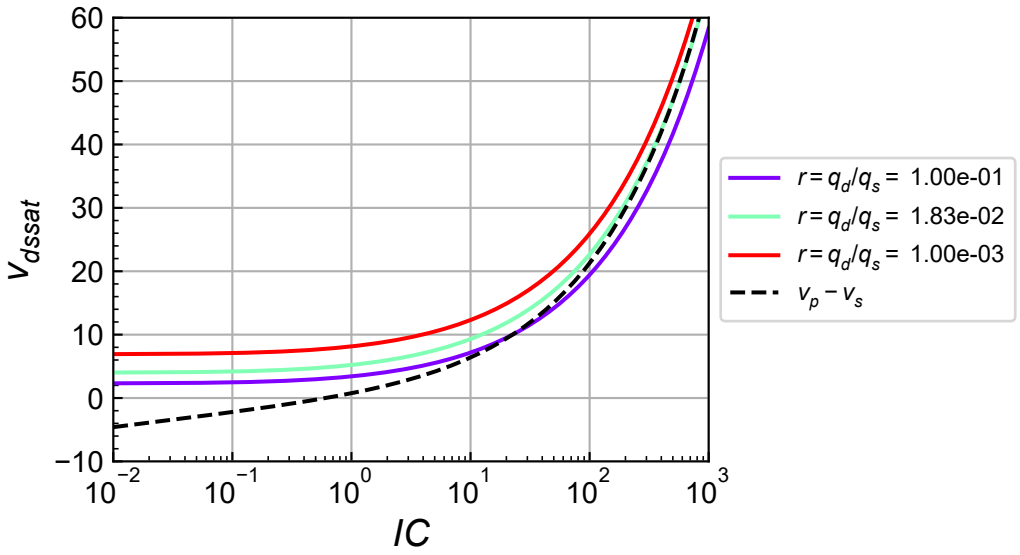

Figure 2.3: v_{dssat} voltage versus IC for various $r = q_d/q_s$ ratioes.

From Figure 2.3, we see that v_{ds} saturates in weak inversion to $v_{ds,wi} \cong -\ln(r)$. The $4U_T$ value that is typically used corresponds to the curve for $r = 1.832\text{e-}02$.

In strong inversion the curve for $r \ll 1$ tends asymptotically to the value of $v_{dssat} \cong 2q_s \cong v_p - v_s$.

The meaning of the $r = q_d/q_s$ ratio might not be very intuitive at first glance and circuit designers prefer to express the saturation voltage in terms of the current ratio $\alpha \triangleq I_R/I_F = i_r/i_f$ [4] [5]

$$v_{dssat} = \left(\sqrt{4\,i_f+1}-1\right)\left(1-\frac{\sqrt{4\,\alpha\,i_f+1}-1}{\sqrt{4\,i_f+1}-1}\right) - \ln\left(\frac{\sqrt{4\,\alpha\,i_f+1}-1}{\sqrt{4\,i_f+1}-1}\right). \tag{2.11}$$

Assuming that $\alpha = i_r/i_f < 1$, the forward current is simply equal to the inversion coefficient $i_f = IC$. Equation (2.11) is plotted versus IC in Figure 2.4 for different values of α.

v_{dssat} in weak inversion tends to

$$v_{dssat,wi} = -\ln\left(\frac{\sqrt{4\,\alpha\,i_f+1}-1}{\sqrt{4\,i_f+1}-1}\right) \tag{2.12}$$

Since in weak inversion $i_f \ll 1$, $\sqrt{4\,\alpha\,i_f+1}-1 \cong 2\,\alpha\,i_f$ and $\sqrt{4\,i_f+1}-1 \cong 2\,i_f$ and hence

$$v_{dssat,wi} \cong -\ln(\alpha) \tag{2.13}$$

or

$$\alpha \cong e^{-v_{dssat,wi}}. \tag{2.14}$$

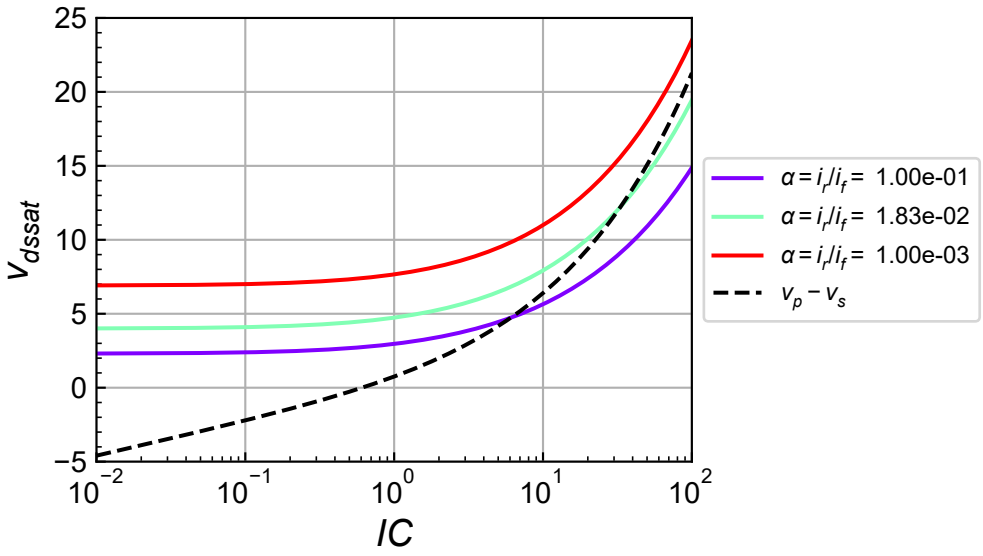

Figure 2.4: v_{dssat} voltage versus IC for various $\alpha = i_r/i_f$ ratioes.

For the usual value of $v_{dssat,wi} = 4$, the ratio $\alpha =$ 1.83e-02, which is equal to the value of $r = q_d/q_s$ used above simply because in weak inversion $q_d \cong i_r$ and $q_s \cong i_f$.

Although the ratio $r = q_d/q_d$ is not very intuitive a priori, it can however be interpreted from a circuit perspective. Indeed, recalling that $q_s = g_{ms}$ and $q_d = g_{md}$, the $r = q_d/q_d$ ratio actually corresponds to the ratio of the drain to the source transconductance

$$r \triangleq \frac{q_d}{q_s} = \frac{g_{md}}{g_{ms}}. \tag{2.15}$$

In the next section, we will show that the ratio $1/r = g_{ms}/g_{md}$ is actually the voltage gain of a common-gate stage.

3 Interpretation of the q_d/q_s ratio

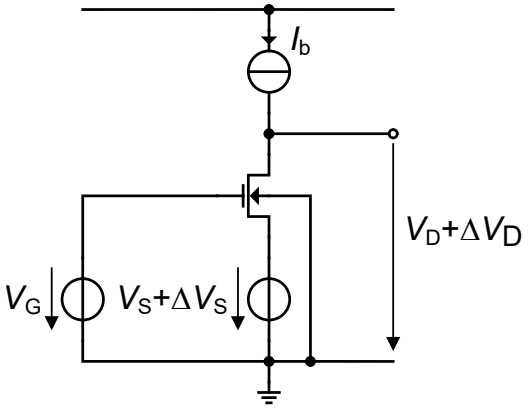

Figure 3.1: Schematic of the common-gate stage .

The circuit of Figure 3.1 represents a common-gate stage biased at a constant current I_b as can be found in a cascode gain stage. If we assume that the current source is ideal and we neglect the output conductance of the transistor, we get the small-signal circuit shown in Figure 3.2.

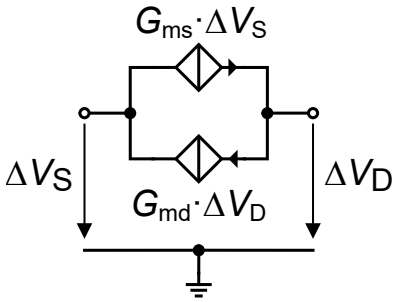

Figure 3.2: Small-signal schematic of the common-gate stage of Figure 3.1.

Since the drain current is maintained constant there can be no change in the drain current leading to [6] [7]

$$\Delta I_D = G_{md} \cdot \Delta V_D - G_{ms} \cdot \Delta V_S = 0. \tag{3.1}$$

The voltage gain from the source to the drain is therefore given by [6] [7]

$$A \triangleq \frac{\Delta V_D}{\Delta V_S} = \frac{G_{ms}}{G_{md}} = \frac{g_{ms}}{g_{md}} = \frac{1}{r}. \tag{3.2}$$

This voltage gain actually corresponds to the inverse of the factor r defined in the previous Section.

We can then rewrite the saturation voltage (2.9) in terms of the voltage gain A as [6] [7]

$$v_{dssat} = 2q_s \left(1 - \frac{1}{A}\right) + \ln(A). \tag{3.3}$$

Equation (3.3) is plotted versus the inversion factor IC in Figure 3.3 for various values of A.

We call also express the drain-to-source saturation voltage in terms of its value in weak inversion $v_{dssatwi}$

$$v_{dssat} = 2q_s \left(1 - e^{-v_{dssatwi}}\right) + v_{dssatwi}. \tag{3.4}$$

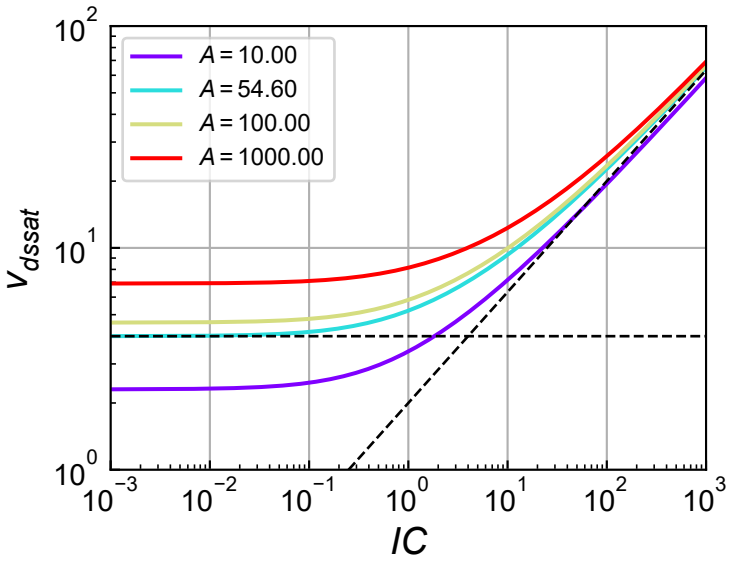

Figure 3.3: v_{dssat} versus IC for different values of A.

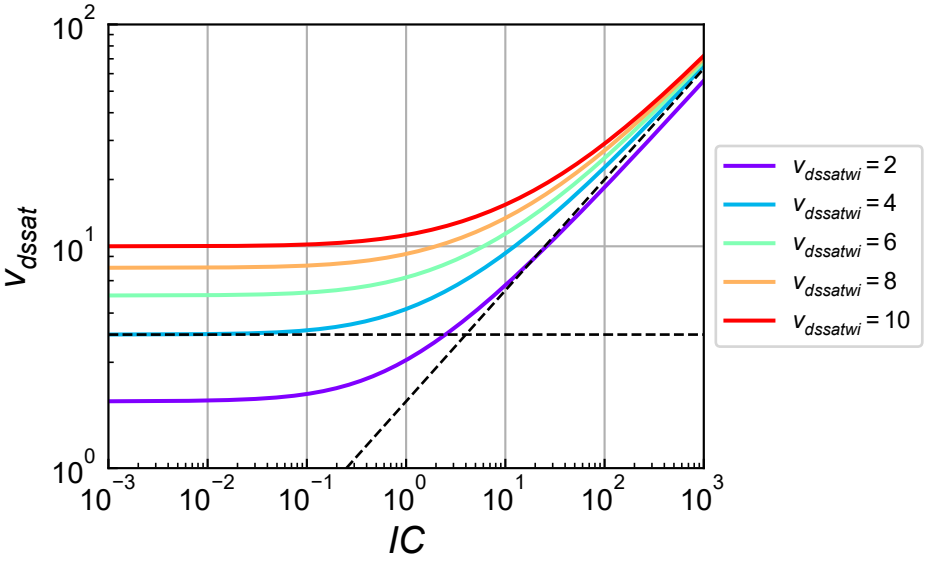

Figure 3.4: v_{dssat} versus IC for different values of A.

The dashed black curves correspond to the weak and strong inversion asymptotes for $v_{dssat,wi} = 4\ U_T$, which corresponds to a voltage gain $A = 54.60$. Therefore a simple approximation of the saturation voltage valid from weak to strong inversion is given by

$$v_{dssat} = 2q_s \left(1 - e^{-4}\right) + 4 \cong \sqrt{4IC + 1} + 3 \tag{3.5}$$

The approximation (3.5) is compared to (3.4) for $v_{dssat,wi} = 4$ in Figure 3.5. We see an almost perfect fit.

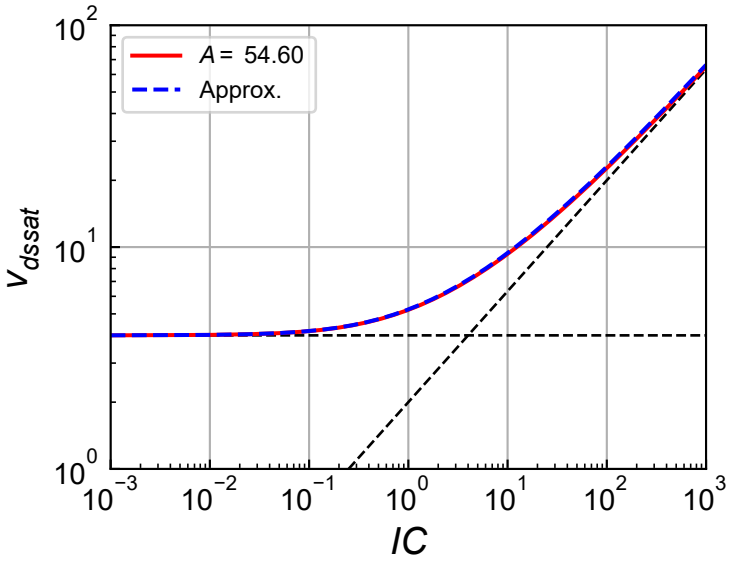

Figure 3.5: Approximation of $v_{ds,sat}$ versus IC.

References

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